Rematching with Contracts under Labor Mobility Restrictions: Theory and Application*

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Abstract

Labor contracts typically do not limit worker mobility. Interesting exceptions exist in foreign worker reemployment, sports transfers and sometimes through non-compete clauses. We develop a model to address contractual designs for such markets. Although legally, a firm can contest its worker's recruitment by a competitor, it may be more lenient if he can be replaced immediately. We develop a theory of stability suitable for such markets and propose stable-uncontested mechanisms. As our application, we consider transfers in collegiate sports governed by the NCAA, where before 2021, a student-athlete had to sit out a year after a transfer. Beginning in 2021, free mobility was allowed. Anecdotal evidence suggests while pre-2021 regulations were detrimental to student and college welfare, post-2020 regulations led to colleges struggling to keep rosters and withholding new scholarship slots to use in transfers. Our model also captures the NCAA's pre-2021 and post-2020 regulations as well as our new proposed efficiency-enhancing criterion. Then, using data from men's collegiate basketball, we estimate college and student-athlete preferences. Using the preferences we estimate from transfer data, we run counterfactual analyses of pre-2021 and post-2020 environments and our proposed regulations. Our proposal achieves closer student-athlete welfare to post-2020 than pre-2021 and increases college welfare with respect to post-2020 and pre-2021.

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1 Introduction

Labor market contracts sometimes involve clauses restricting a worker's recruitment by a competitor. For example, non-compete clauses limit the employment of a high-skilled worker by a competitor within a period after resignation or termination. They are used in different sectors to protect trade secrets acquired during the worker's tenure at a firm. There are also other reasons for mobility restrictions. Workers cannot usually change jobs freely in a foreign country due to immigration laws, sometimes aimed at justifying the search and transaction costs of the foreign firm in initially hiring them. Many professional sports leagues worldwide have had practices that restricted not only players freely changing teams but also free agency after the ending of such contracts. Some of these restrictions were eventually lifted at least regarding free agency. However, some still exist. For instance, in the US National Football League (NFL) and the US National Basketball Association (NBA), the existing teams have the right to keep the restricted free agents by matching the offer proposed by other teams.[1]

Although some limitations may be necessary for specific reasons, as outlined above, economists and policymakers typically think severe restrictions are detrimental to the welfare of the workers, human rights, and social welfare in general due to anti-competitive practices. Indeed, labor laws in recent decades, to a large extent, evolved toward fewer restrictions.[2]

Severe restrictions and no restrictions regarding worker mobility are easy to implement in a market place. However, using less restrictive clauses for societal gain is not straightforward.

In this paper, we use the evolution of a unique market's contractual agreement structure to measure the welfare effects of fully restrictive and free labor mobility on market participants. We introduce a new contractual instrument design that mediates unfairness of resulting outcomes as a compromise solution and alleviates the inefficiency observed under these two extreme institutions. To this end, we first develop a new theoretical model and framework using matching theory. We use this framework to develop a new contractual instrument that allows an intermediate level of contractual restrictions, not as permissive as free mobility but also not as prohibitive as severe restrictions. We also use this design in a counterfactual empirical analysis to measure its possible effects on welfare and fairness.

Application: The US College Student-Athlete Transfer Market. The unique labor market we study involves the retention and transfer practices of colleges for their varsity sports teams in the US. The American college sports landscape is unique in the world as it tries to accomplish multiple goals at once. The players in a college's sports teams are students who attend

[1] See https://www.nfl.com/news/nfl-free-agency-frequently-asked-questions-0ap1000000148425, https://www.nba.com/news/free-agency-explained. All internet links provided in the paper are current as of November 25, 2025.

[2] For example, the US Federal Trade Commission (FTC) even tried banning all non-compete clauses, unsuccessfully. As of the writing of this draft, FTC does not plan to enforce any further such bans.

the college, often receiving athletic scholarships that cover, in full or in part, a very expensive college education. The primary goal is squarely set by the colleges as providing these students with higher education. For a majority of the student-athletes, this goal indeed overlaps with theirs for participating in college sports. However, the most competitive college teams and their respective conferences,[3] which are the primary drafting ground for several professional sports in the US, college sports showcase talented student-athletes who eventually turn professional. Such colleges gain prestige, monetary income, and academic competitiveness by recruiting or developing these students into star athletes.[4] As a result, these dual goals may sometimes conflict with each other.

In the backdrop of this landscape, transfers of student-athletes between colleges are regulated by the National Collegiate Athletic Association (NCAA). Before 2021, student-athletes who transferred from one college to another generally had to *wait out* one year before becoming eligible to play in their new team.[5] Consequently, the transfer restrictions gave teams and their coaches much control over the student-athletes. Eventually, the NCAA lifted all restrictions.[6] Colleges that lose a player often need to seek a new student-athlete via transfer or recruit a new high school graduate in the upcoming recruitment season. Thus, waiting out a year was usually perceived as a regulatory tool to discourage transfers, akin to non-compete clauses in high-skill labor markets, visa restrictions for foreign labor, or free-agency or transfer restrictions in professional sports.

However, neither full restrictions with a penalty of waiting out a year nor free transfers without any penalty may be desirable. Under full restrictions, student-athletes who would like to attend and play at a different college immediately are penalized. However, a college that loses the student-athlete may not mind, if it can immediately transfer an eligible player as a substitute.

On the other hand, under free transfers, many colleges find themselves in a player shortage. Therefore, the free transfer regime may discourage colleges from recruiting often overlooked,

[3]Many colleges with varsity sports are grouped in conferences that compete in all sports with each other in their respective leagues.

[4]Due to the vast geography of the country and location of high-level professional teams mostly in relatively large metropolitan areas in any given sport, these college teams are often the main sports attraction in their respective regions with a huge fan-base and, sometimes, national following, due to large alumni population of big colleges. This translates to large revenues for college athletic divisions due to lucrative broadcasting deals of their respective conferences. While most of the revenue is used in subsidizing less lucrative sports programs and their student-athletes, the star players do not receive explicit monetary compensation from the college except for the standard athletic scholarships, while outside compensation became legal recently. Also, success in national collegiate sports often turns into academic selectivity for a college, as more students apply due to its heightened visibility. It may also receive more donations that increase the financial security and academic profile of the school.

[5]The student-athlete sometimes appealed the decision to become immediately eligible. In certain instances, these appeals were approved, and others were not.

[6]The NCAA also lifted restrictions on student-athletes regarding earning income through advertising or marketing activities. Some viewed the transfer restrictions through the same lens. Increasingly, the recent attitude of the NCAA has been shifting toward recognizing student-athletes more as athletes. Judicial and legislative processes in some US States have also nudged the NCAA to move in this direction in recent years.

high-potential high-school recruits who can leave them in the future, adversely affecting the future of such talent and overall market efficiency. In addition, colleges reserve some of their scholarships for transfers, decreasing the number of scholarships available for high school seniors.[7]

A Possible Remedy and the Model. Instead, we propose a contractual structure milder than complete curtailment of immediate eligibility of new transfers and more restrictive than free transfers.

What if the immediate eligibility of a transfer student-athlete were granted as long as his initial college replaced him with a new transfer who does not have to wait out one year? Our introduced regulatory environment addresses this scenario. To use the simplest possible setup to capture it, we consider a two-sided matching model with at most two contractual terms that can be signed between a college and a student-athlete: the student-athlete is immediately eligible to play (I) or must wait out a year (W). We assume initially, each student-athlete plays on a college team. In the transfer market, colleges try to get new student-athletes, and student-athletes look for colleges to transfer to.

This remedy is not only compatible with both older and current regulations to some degree, but also incorporates features from both regimes, with the simplest possible departure from both. We also capture the old and the new NCAA environments by endowing some colleges the right to block certain transfers of their players. We refer to such colleges as *restrictive colleges*. Transfers are allowed in two forms:

1. A student-athlete can freely transfer to a different college with a contractual term of waiting out a year (W); or
2. a student-athlete can transfer to a different college with immediate eligibility (I) if
    - his initial college is not a restrictive college, or
    - his initial college is a restrictive college and can also transfer a player with immediate eligibility (I) to replace him.

Thus, our model captures the regulatory environment in different NCAA eras and our new proposed design:

- If contractual term I is unavailable for any transfer, this setting corresponds to the pre-2021 NCAA environment.
- If no college is restrictive and contractual term I is available for all transfers, this setting corresponds to the post-2020 NCAA environment.
- Our new proposed contractual design refers to a setting where contractual term I is available for all transfers, and colleges are restrictive.

[7]See https://www.startribune.com/transfer-portal-high-school-athletes-recruiting/600188986/?refresh=true and https://www.wbrc.com/2021/12/13/is-college-transfer-portal-hurting-high-school-students-opportunity-athletic-scholarship.

To define our solution concept compatible with our contractual framework, we introduce new axioms:

In a given matching, for a restrictive college, losing student-athletes with term-I contracts is *uncontested* if it also signs an equal number of term-I contracts with incoming transfer student-athletes. If all contractual assigments are uncontested by restrictive colleges then this matching is uncontested.

An *individually rational* matching assigns each student-athlete a contract that is at least as good as staying at his initial college with term-I contract and each college with only acceptable contracts.[8]

A matching is *stable* if it is uncontested, satisfies individual rationality, and involves no blocking college and student-athlete pair such that their match would have led to an uncontested matching when other contracts stay the same.

A matching is *constrained efficient* if there does not exist another uncontested matching that matches each student-athlete with the same college but possibly with different contractual terms and Pareto dominates this matching.

The pre-2021 NCAA regime can be modeled by term-I contracts being infeasible for transfers. In this case, there are no contractual externalities caused by incontestability. Stability is equivalent to a modification of the standard stability notion, which gives student-athletes the right to stay in their initial colleges. This is exactly the stability notion of Gale and Shapley (1962) applied for our model with the auxiliary modification to the college preferences to rank their initial players ahead of any other student-athlete from a different college. In this case, the student-optimal stable mechanism of the auxiliary market is strategy-proof for student-athletes, stable, and Pareto efficient. We refer to this mechanism as *Cumulative Offer Process without Immediately Eligible Transfers* (*COP-no-I*).

In the post-2020 NCAA regime, no college is restrictive, and term-I contracts are feasible. There are no contractual externalities caused by incontestability in this case, either. Stability is equivalent to a modification of the standard stability notion in the "matching with contracts model" of Hatfield and Milgrom (2005) with the auxiliary modification that gives student-athletes the right to stay in their initial colleges with contract term I. A mechanism that satisfies strategy-proofness for students, stability, and constrained efficiency exists. It is the *Cumulative Offer Process with Unrestrictive Colleges* (*COP-Unr*) and obtained by modifying first the preferences of colleges to make sure that their initial students get the highest rank with contract term I and then executing the *Cumulative Offer Process* (*COP*) of Hatfield and Milgrom (2005) (cf. Hatfield and Kojima, 2010).

[8]Thus, each college considers its initial players acceptable with contract term I and each student-athlete finds his initial college acceptable with contract term I.

A New Mechanism and Theoretical Results. Contractual externalities only matter in our designed environment when colleges are restrictive and term-I contracts are feasible. We show that a student-strategy-proof and stable mechanism exists. However, with this mechanism, all transfers are with contract term W. We also show that stability and constrained efficiency are incompatible when colleges are restrictive.

However, we can still use contracts with term I to obtain welfare gains, although a constrained-efficient and stable mechanism does not exist. To this end, we introduce a new mechanism that is stable and enables students to transfer with contractual term I: the *Uncontested Cumulative Offer Process* (*UCOP*). Intuitively, UCOP limits the number of term-I contracts that a college can be assigned.

To do so, we devise an auxiliary choice rule to ensure that the number of *tentatively* imported students with term I never exceeds the number of initial students at each college, who, by definition, have term-I contracts with the college.

After running COP using this auxiliary choice rule, UCOP's intermediate outcome may violate stability through a contesting college or a blocking pair that would lead to an uncontested outcome when satisfied. UCOP handles these issues by updating preferences accordingly, and the process is repeated until UCOP obtains a stable and, therefore, uncontested outcome.

In its final stage, UCOP looks for welfare-improving cycles while respecting stability and without changing the assigned colleges of the students. That is, UCOP looks to improve welfare on both sides of the market by changing contracts in the final outcome from W to I while maintaining the balance of term-I contracts for colleges.

We show that when colleges are restrictive, UCOP is stable but neither strategy-proof nor constrained efficient. However, among the set of uncontested (and therefore stable) mechanisms, any possible welfare improvement over UCOP leads to a tradeoff in properties. Specifically, (1) there is no uncontested and strategy-proof mechanism that Pareto dominates UCOP, and (2) any mechanism that Pareto improves upon UCOP, whenever possible, cannot be less manipulable than UCOP.

Finally, we show the compatibility among several good properties in a case where the market starts in a situation where there are no initially vacant positions at any college. This case allows us to propose a simplified version of UCOP and show that it is strategy-proof for students, stable, constrained efficient, and all transfers are with contract term I.[9]

Empirical Analysis: Identification Methodology and Estimation. Using a unique data set on student-athlete transfers involving Division I men's college basketball, the highest level of collegiate competition, we analyze the welfare effects of this proposed regulation as well as the

[9]Interestingly, we also show that in this case if additionally, each school has a single initial student-athlete, *Top Trading Cycles with Contracts* mechanism that we introduce satisfies all desirable properties: Pareto efficiency, stability, and strategy-proofness for students.

old and new market rules. This is a relatively large market: there were 5,485 student-athletes on men's basketball Division I rosters in 2022, where 1,649 students declared an intention to transfer and 1,123 students transferred. We use an econometric model to estimate preferences and demonstrate possible improvements with the tools from market design. We find substantial welfare improvements in our mechanism using a counterfactual analysis. We estimate students' preferences over colleges and colleges' preferences over students using initial recruitment data of high school students.

The key component of our econometric identification is the assumption of the stability of the observed outcome matching. Although Fack, Grenet, and He (2019) introduced the idea of using stability for the identification of complete preferences for students, it applied to centralized one-sided markets with preset priorities for colleges. In contrast, we have a decentralized two-sided matching market, where we estimate both sides' preferences using stability as a market equilibrium concept.

For students' preferences, the variation in the data that identifies preferences is the observable characteristics of colleges that predict which offer the student will accept, conditional on the set of offers he received. For students, we know a student's set of offers and which college they chose. The behavioral requirement implied by an assumption of stability is as follows: students must choose their most preferred college from among those that offered them admission.

For colleges' preferences, the variation in the data that identifies preferences is the observable characteristics of students that predict which students a college will make an offer to. For colleges, unlike for students, we do not observe choice sets and instead know only the universe of students to whom they could make an offer. We address this empirical challenge with an innovative approach for constructing colleges' choice sets. The behavioral requirement implied by the assumption of stability is as follows: each college must prefer every student it made an offer to over every student who did not receive an offer but would have accepted if offered. Once colleges' choice sets are constructed, the behavioral requirement implied by an assumption of stability is as follows: colleges must make an offer to the most preferred students in the constructed choice set.

Counterfactual Analysis. Using the estimates from our preference estimation, we conduct a series of counterfactual analyses that allow us to simulate the NCAA transfer market with different transfer rules. We compare the UCOP outcome to two other mechanisms. The *Before mechanism* implements a simplified version of the NCAA transfer rules before the policy changes in 2021. Prior to 2021, it was difficult for students to transfer with immediate eligibility. We use the COP-no-I mechanism to simulate the equilibrium in the decentralized pre-2021 era transfer market without term-I transfers. The *After mechanism* implements a simplified version of the NCAA transfer rules following the 2021 policy changes. Although this is a decentralized market as well, we use the COP-Unr outcome to emulate the market equilibrium. Finally, the UCOP mechanism implements our regulatory proposal.

The counterfactual results are presented for three sets of outcomes of interest: (1) transfer outcomes, (2) students' preferences between mechanisms, and (3) colleges' imbalance. Transfer outcomes are either transfer with contractual term I, transfer with term W, or no transfer. Students' preferences are pairwise comparisons between mechanisms (Before, UCOP, and After) asking whether a student prefers their assignment from one mechanism to their assignment from another mechanism. College imbalance measures unfilled positions in a college's roster. Imbalance is measured in two ways: *Overall imbalance* measures the number of colleges in which the number of imports is less than its quota and *short-term imbalance* measures the number of colleges in which the number of imports with term I is less than its quota.

The results show that UCOP performs similarly to the After mechanism with respect to transfer outcomes in general and transfers with term-I contracts specifically. In terms of students' preferences, very few students prefer Before to other mechanism outcomes. Relative to After, between 30% and 60% of students strictly prefer After to UCOP, with the remaining students receiving the same assignments in After and UCOP. Turning to colleges, we use imbalance to measure college welfare. UCOP is much better than the other two mechanisms in terms of minimizing imbalance. UCOP features very little short-term imbalance: fewer than 10% of colleges have overall imbalance under UCOP and overall imbalance with UCOP is around 0% and 1% in 2013 and 2016, respectively. The Before mechanism is worst at minimizing short-term imbalance, while the After mechanism is worst at minimizing overall imbalance. Overall, the results are supportive of our arguments that UCOP works well as a compromise mechanism that leads to fairer outcomes than the other two regulatory environments.

2 Model

We introduce a model of student-athlete transfers within a matching with contracts framework. Our model fits a number of interesting settings, but for concreteness, we use college and student to refer to agents on the two sides of the market.[10] The contractual structure of interest differentiates between agents who sign a more flexible contract relative to those who sign a less flexible contract. The specific terms we use are contracts with immediate eligibility (term-I contracts) relative to contracts requiring waiting out a year before playing (term-W contracts). These phrases are drawn from the setting in which we will conduct our empirical analysis, which is transfers of student-athletes governed by the National Collegiate Athletic Association (NCAA) in the US.

2.1 Basics

We consider a two-sided market composed of colleges and students. Let C be the finite set of **colleges** and S be the finite set of **students**. We define function $\omega : S \to C$, where $\omega(s)$ is the

[10]We discuss these other settings in Section 7.2.

initial college of student $s \in S$. Let $S_c = \{s \in S : \omega(s) = c\}$ for each $c \in C$ and $\bigcup_{c \in C} S_c = S$ — so that each student is currently assigned to some college. We consider a *matching with contracts* framework introduced by Hatfield and Milgrom (2005). Let $X = (S \times C \times T) \cup \{x_\emptyset\}$ be the set possible **contracts**, where the set of **contractual terms** (or simply **terms**) is given as $T = \{I, W\}$ and $x_\emptyset$ is null contract. Term I refers to a transfer with immediate eligibility so that the transferred student can play at the new college without waiting out. Term W refers to a transfer without immediate eligibility so that the transferred student must wait out for one year before playing at the new college. Contract $x = (s, c, t)$ means student s signs contract with college c with term t. Let $\mathrm{s}(x)$, $\mathrm{c}(x)$, and $\mathrm{t}(x)$ be the student, college, and term related to the contract x. For any $Y \subseteq X$, let $\mathrm{s}(Y) = \bigcup_{x \in Y} \mathrm{s}(x)$. For any $Y \subseteq X$, let Y_s and Y_c be the sets of contracts in Y that are related to student s and college c, respectively. Similarly, let Y_t be the sets of contracts in $Y \subseteq X$ containing contract term t. Each student s is initially assigned to his initial college $\omega(s)$ with term I — a student who does not transfer can play immediately at his initial college. We refer to $(s, \omega(s), I)$ as the **initial contract** of student s.

The administration of a college c might give it the power to control the contracts signed by its initial students, and college c might seek to exert this power. Alternatively, the governing policymaker (e.g., the NCAA) might give the college c this power to control. We refer to such colleges as **restrictive colleges** and denote the set of restrictive colleges with C^R. If a college is not restrictive, then we call it an **unrestrictive college**.

Each student s has **strict preferences** over contracts in $X_s \cup \{x_\emptyset\}$ and each college c has strict preferences over contracts in $X_c \cup \{x_\emptyset\}$. Let P_s and P_c denote the (strict) preferences of student s and college c over the contracts in X_s and X_c, respectively.[11] Their induced weak preference relations are denoted as R_s and R_c, respectively.[12] We say a contract x is **acceptable** for agent $i \in S \cup C$ if $x \; P_i \; x_\emptyset$. Otherwise, contract x is **unacceptable** for agent i. Let q_c be the quota of college c such that $q_c \geq |S_c|$, i.e., each college c has enough capacity for its initial students. Let $q = (q_c)_{c \in C}$.

Each college c has a choice rule over subsets of contracts which is induced by its preferences over individual contracts. Formally a **choice rule** is a mapping $\mathcal{C}_c : 2^X \to 2^{X_c}$. We assume that college choice rules are *responsive* according to the following definition. A choice rule $\mathcal{C}_c$ is **responsive to preferences over individual contracts** (or simply **responsive**) if given a subset of contracts $X' \subseteq X$, the chosen set of contracts, denoted by $\mathcal{C}_c(X')$, is determined through the following iterative procedure:

Step 0: Set $X^1 := \{x \in X' \cap X_c \; : \; x \; P_c \; x_\emptyset\}$, and initialize $Y^1 := \emptyset$ and $q_c^1 := q_c$. If $X^1 = \emptyset$ or $q_c^1 = 0$, then the procedure terminates with $\mathcal{C}_c(X') = Y^1 = \emptyset$. Otherwise, we continue with Step 1.

[11] We assume preferences over individual contracts are strict, meaning that they are linear orders or binary relations that are complete, antisymmetric, and transitive.

[12] That is, for each agent i and contract pair $x, y \in X_i$, $x \; R_i \; y$ if and only if $x = y$ or $x \; P_i \; y$.

Step k: (k > 0) Let

$$x_k := \max_{P_c} X^k.$$

We define

$$Y^{k+1} := Y^k \cup \{x_k\}, \quad X^{k+1} := X^k \setminus X'_{\mathrm{s}(x_k)}, \quad \text{and} \quad q_c^{k+1} := q_c^k - 1.$$

If $q_c^{k+1} = 0$ or $X^{k+1} = \emptyset$, then the procedure terminates with $\mathcal{C}_c(X') = Y^{k+1}$, otherwise we continue with Step k+1.

This concept is an extension of responsive (Roth, 1985) and separable preferences with fixed (or no) contractual terms in the matching literature.

Next, we define other properties of choice rules we use in our analysis. A choice rule $\mathcal{C}_c$ is **substitutable** (Kelso and Crawford, 1982; Roth, 1984b) if whenever $x \notin \mathcal{C}_c(X' \cup \{x\})$ for some $X' \subset X$ and $x \in X$, then $x \notin \mathcal{C}_c\big(X' \cup \{x, x'\}\big)$ for any $x' \in X$. A choice rule $\mathcal{C}_c$ satisfies the **law of aggregate demand** (**LAD**) (Alkan and Gale, 2003; Hatfield and Milgrom, 2005) if $X' \subset X''$ implies $|\mathcal{C}_c(X')| \leq |\mathcal{C}_c(X'')|$. A choice rule $\mathcal{C}_c$ satisfies **independence from rejected contracts** (**IRC**) (Blair, 1988; Aygün and Sönmez, 2013) if whenever $x \notin \mathcal{C}_c(X')$ then $\mathcal{C}_c\big(X' \setminus \{x\}\big) = \mathcal{C}_c(X')$. In Appendix A, Lemma 1 shows that a responsive choice rule is substitutable and satisfies both LAD and IRC.

For ease of exposition, assume that each college c has a preference relation over groups of contracts that is consistent with its choice rule $\mathcal{C}_c$. That is, for any $X' \subseteq X_c$, college c prefers $\mathcal{C}_c(X')$ to any $X'' \in 2^{X'} \setminus \big\{\mathcal{C}_c(X')\big\}$. Let P_c also denote this preference relation with a slight abuse of notation.

An outcome of a problem is a matching. A **matching** μ is a set of contracts such that

(i) $|\mu_s| \leq 1$ for all $s \in S$, and

(ii) $|\mu_c| \leq q_c$ for all $c \in C$.

If $\mu_i = x_\emptyset$ for some $i \in S \cup C$, then i is unassigned under matching μ.[13]

Matchings μ and ν are **partner equivalent** if $\mathrm{c}(\mu_s) = \mathrm{c}(\nu_s)$ for every $s \in S$.[14]

Under matching μ, for each college $c \in C$, the set of contracts of students in S_c who are matched to colleges other than c is denoted by μ_c^e, i.e.,

$$\mu_c^e = \Big(\bigcup_{s \in S_c} \mu_s\Big) \setminus \big(\mu_c \cup \{x_\emptyset\}\big).$$

We refer to μ_c^e as the **exports** of college c. Similarly, under matching μ, for each college $c \in C$,

[13] Null contract $x_\emptyset$ is also identical to the empty set for contract sets by slight abuse of notation, thus for any agent i, $Y_i = Y \cap X_i = \emptyset$ and $Y_i = x_\emptyset$ are identical statements for any $Y \subset X$ including when Y is a matching.

[14] Notice that $\mathrm{c}(\mu_s) = \mathrm{c}(\nu_s)$ for every $s \in S$ implies $\mathrm{s}(\mu_c) = \mathrm{s}(\nu_c)$ for every $c \in C$.

the set of contracts assigned to c of students in $S \setminus S_c$ with μ_c^m,

$$\mu_c^m = \mu_c \setminus \Big(\bigcup_{s \in S_c} \mu_s \cup \{x_\emptyset\} \Big).$$

We refer to μ_c^m as the **imports** of college c.

2.2 Properties

In this environment, we define several desirable properties of matchings, some novel to our problem, and others standard. We start with novel properties. We assume that in a transfers market, a student requires the permission of his college to move and play immediately if his college is a *restrictive* college.[15] Formally, a student $s \in S$ can sign a contract with term W freely and with term I freely if only his initial college is not restrictive, i.e., $\omega(s) \in C \setminus C^R$. On the other hand, a student s can sign a contract $x = (s, c', I)$, where $c' \neq \omega(s)$ and $\omega(s) \in C^R$ if he is allowed to by his initial college $\omega(s)$. We incorporate this feature into our model using the following notion: for each restrictive college c, there cannot be more exports than imports with term I. Notice that, having more exports than imports might cause roster problems.

We say matching μ is **uncontested by college** c if

$$\left|\mu_c^e \cap \mu_I\right| - \left|\mu_c^m \cap \mu_I\right| \leq 0.$$

We refer to this property as **incontestability**. That is, the difference between the numbers of exports and imports with term I is less than 0. A matching μ is **uncontested** if it is uncontested for all restrictive colleges.

For the rest of our analysis, we consider two cases: either all colleges are restrictive, or all colleges are unrestrictive.[16] One can easily see that when colleges are restrictive, in an uncontested matching μ, $\left|\mu_c^e \cap \mu_I\right| = \left|\mu_c^m \cap \mu_I\right|$ for every $c \in C$. Moreover, when colleges are unrestrictive, any matching is uncontested.

We illustrate incontestability through a simple example when colleges are restrictive.

Example 1 *There are three colleges, $C = \{a, b, c\}$, and two students, $S = \{s, s'\}$. Let $S_a = \{s\}$, $S_b = \{s'\}$, $S_c = \emptyset$. Colleges are restrictive so that $C^R = C$. Each college has one seat. Suppose all contracts are acceptable for all agents. Any matching in which both students are assigned with term W is uncontested. Considering the other matchings in which both students are matched, the other uncontested matchings are:*

[15] As explained in the introduction, when restrictions are applied, transfers with immediate eligibility are permitted case-by-case through an opaque process. Our mechanism formalizes this process to make it transparent while increasing efficiency.

[16] In Appendix B, Example 4 shows that the desirable properties defined in this section cannot hold when we consider an environment with both restrictive and unrestrictive colleges.

$$\mu^1 = \{(s,a,I),\ (s',b,I)\},$$
$$\mu^2 = \{(s,b,I),\ (s',a,I)\},$$
$$\mu^3 = \{(s,a,I),\ (s',c,W)\},$$
$$\mu^4 = \{(s,c,W),\ (s',b,I)\},$$
$$\mu^5 = \{(s,a,I),\ (s',b,W)\},$$
$$\mu^6 = \{(s,a,W),\ (s',b,I)\}.$$

In Example 1, one can notice that no student is assigned to c with a contract with term I in any uncontested matching. Here, college c starts with fewer initially assigned students than its quota. This observation holds in general.

Observation 1 *When colleges are restrictive, in any uncontested matching μ, the number of contracts with term I assigned to a college c cannot exceed the number of students in S_c, that is, $|S_c| \geq |\mu_c \cap \mu_I|$.*

For a student s to move to another college, the contract he will sign should be better than his initial contract $(s, \omega(s), I)$. On the other hand, after all transfers are realized, a college c may end up with a set of contracts worse than its initial set of contracts. Based on these observations, we say that a matching μ is **individually rational** if no student s is assigned a contract worse than his initial contract $(s, \omega(s), I)$ and no agent $i \in S \cup C$ is assigned an unacceptable contract.

Under the current regulations of the NCAA, enacted in 2021, students do not need the permission of their initial colleges to move. Restrictions may only apply for immediate eligibility when colleges are restrictive, as in the pre-2021 period. Thus, for any axiom to be relevant in our model, it must be compatible with a corresponding decentralized market equilibrium concept. To this end, we take stability as an important axiom.[17] Differently from the standard two-sided matching markets, e.g., the college admission problem, here incontestability introduces indirect externalities to the market. Hence, our definition of stability needs to account for incontestability. We say a student-college pair (s,c) **standard-blocks** a matching μ with contract term t if

$$(s,c,t)\ P_s\ \mu_s \quad \text{and} \quad (s,c,t) \in \mathcal{C}_c\big(\mu_c \cup \{(s,c,t)\}\big).$$

We say a student-college pair (s,c) **blocks** a matching μ with term t if

(i) (s,c) standard-blocks μ,

(ii) there does not exist $s' \in S_c$ such that

$$(s',c,I) \in \mu_c \setminus \mathcal{C}_c\big(\mu_c \cup \{(s,c,t)\}\big), \text{ and}$$

[17] See Roth (1984a) and Roth (1991) for practical motivation regarding stability being an important contributor to the success of a real-life mechanism that is implemented in a previously decentralized market. From a theoretical point of view the set of stable matchings, as defined by Gale and Shapley (1962), is equivalent to core of the market, a solution concept that is related to competitive equilibrium (Herings, 2024).

(iii) if $t = I$, then the matching

$$\nu = \big(\mu \setminus (\mu_c \cup \mu_s)\big) \cup \mathcal{C}_c\big(\mu_c \cup \{(s,c,t)\}\big)$$

is uncontested.

Here matching ν is obtained from μ by **satisfying** the blocking pair (s,c) with term t, that is by adding (s,c,t), removing contracts μ_s and $\mu_c \setminus \mathcal{C}_c\big(\mu_c \cup \{(s,c,t)\}\big)$, and keeping the matches of other colleges and students under μ. Part (ii) is a natural individual rationality assumption for students: a student always has the right to stay at his initial college if he chooses. Formally, a standard-blocking pair should not cause the rejection of any term-I contract involving the college's initial students. Part (iii) is also a natural condition in our setting: when we satisfy a standard-blocking pair with term I, the matching obtained should be uncontested. Notice that Part (iii) becomes redundant when colleges are unrestrictive since every matching is uncontested.

A matching μ is **stable** if it is uncontested, individually rational, and there is no blocking pair.[18] This definition is different from the standard stability definitions of Gale and Shapley (1962) and Hatfield and Milgrom (2005) as our market involves externalities. Instead of incorporating externalities into the preferences of colleges, we model them through incontestability requirements.

In our welfare analysis, we compare a given matching with uncontested ones. A matching μ is **Pareto efficient** if there does not exist another uncontested matching ν such that all agents are weakly better off under ν compared to μ and at least one agent is strictly better off. A matching μ is **constrained efficient** if there does not exist another uncontested matching ν that is partner equivalent to μ such that all agents are weakly better off under ν compared to μ, and at least one agent is strictly better off. Pareto efficiency implies constrained efficiency. A stable matching μ is **undominated stable** if there does not exist another stable (and therefore uncontested) matching ν such that all agents are weakly better off under ν compared to μ and at least one agent is strictly better off.

In the rest of our analysis, we make the following mild assumption regarding preferences between contracts with terms I and W.

Assumption 1 *For each student s and college c,*

(i) Immediate eligibility is always better than waiting out for student s given that he is matched with college c; that is,

$$(s,c,I)\ P_s\ (s,c,W).$$

[18] First notice that when colleges are unrestrictive, stability only requires individual rationality and elimination of blocking pairs. In Appendix B, Example 5 shows that when colleges are restrictive if we require the elimination of blocking coalitions including multiple colleges and students, then the existence of a stable matching cannot be guaranteed.

(ii) The initial contract between student s and his initial college $\omega(s)$ is always acceptable for both; that is,

$$\big(s,\omega(s),I\big)\ P_s\ x_\emptyset \quad \textit{and} \quad \big(s,\omega(s),I\big)\ P_{\omega(s)}\ x_\emptyset.$$

(iii) The preference relation over contracts with different terms are consistent with respect to the matched partners both for student s and college c as follows: For each student s' and college c',

$$\begin{aligned}(s,c,I)\ P_s\ (s,c',I) &\iff (s,c,W)\ P_s\ (s,c',W), \textit{ and}\\ (s,c,I)\ P_c\ (s',c,I) &\iff (s,c,W)\ P_c\ (s',c,W).\end{aligned}$$

(iv) If the term-W contract with student s is acceptable for college c, then so is the term-I contract, that is

$$(s,c,W)\ P_c\ x_\emptyset \implies (s,c,I)\ P_c\ x_\emptyset.$$

Assumption 1 is straightforward. Observe that our assumption does not restrict a college's preferences over contracts with immediate eligibility and waiting out involving a student with a different initial college. From now on we assume preferences satisfy Assumption 1.

We denote a **market** by the preference profile of students and colleges $P = (P_i)_{i \in C \cup S}$. A **mechanism** ψ is a function that maps any given market P to a matching $\psi(P)$.

A mechanism ψ is **strategy-proof for students** (or **student-strategy-proof**) if no student can gain from misreporting his preferences over contracts, that is, for every student s, market P, and preference relation P'_s,

$$\psi_s(P_s, P_{-s})\ R_s\ \psi_s(P'_s, P_{-s}).$$

A mechanism ψ satisfies a property of matchings if, for each market P, its outcome $\psi(P)$ satisfies that property in P.

3 Proposed Solutions

In this section, we introduce stable mechanisms when colleges are restrictive or when colleges are unrestrictive. When colleges are unrestrictive, the initial assignment does not lead to an implicit externality. As a result, we can achieve good efficiency and strategy-proofness properties under our proposed mechanism while enabling transfers with immediate eligibility. However, when colleges are restrictive, the initial assignment leads to an implicit externality, and thus, achieving all desired properties is challenging. Nevertheless, when colleges are restrictive, our proposed mechanism achieves desired properties under certain conditions, and any possible welfare improvement over our proposed mechanism is costly. Sections 3.1 – 3.3

focus on a transfer market where colleges may have vacant positions initially, while Section 3.4 considers the case where no college starts with a vacant position.

3.1 Preliminary Results and Cumulative Offer Process with Unrestrictive Colleges

We first define the Cumulative Offer Process (COP) (Hatfield and Milgrom, 2005), the version of which we use in this paper is a variation of Gale and Shapley (1962) student-proposing deferred acceptance algorithm due to Roth (1984b) with substitutable college preferences in existence of contractual terms. This version is the basis for the mechanisms we introduce in this section and our main mechanism that is introduced in Section 3.3.

Cumulative Offer Process (COP):

For a student preference profile P_S and college choice rules $(\mathcal{C}_c)_{c\in C}$, COP selects its outcome following these steps:

Step 1: Each student s proposes his best contract x under P_s to college $\mathrm{c}(x)$. Let X_c^1 be the set of contract offers received by each college c in Step 1. Each college c rejects the contracts in $X_c^1 \setminus \mathcal{C}_c(X_c^1)$.

If there are no rejections, COP terminates with the outcome matching $\mu = \bigcup_{c\in C} X_c^1$. Otherwise, we continue with Step 2.

In general,

Step k: (k $>$ 1) Each student s proposes his best contract x under P_s that has not been rejected yet to college $\mathrm{c}(x)$. Let X_c^k be the set of contract offers received by each college c in Step k. Each college c rejects the contracts in $X_c^k \setminus \mathcal{C}_c(X_c^k)$.

If there are no rejections, COP terminates with the outcome matching $\mu = \bigcup_{c\in C} X_c^k$. Otherwise, we continue with Step k+1.

COP will be a fundamental part of most of the mechanisms we propose. We start with two preliminary mechanisms.

When colleges are unrestrictive, we propose a mechanism, whose outcome is found using the following procedure for any given market P:

Cumulative Offer Process with Unrestrictive Colleges (COP-Unr):

Construct a preference relation $\widehat{P}_c$ from P_c for each college c as follows: Move contract (s, c, I) for all $s \in S_c$ above all other contracts while maintaining the relative order of the other contracts. Let $\widehat{\mathcal{C}}_c$ be the choice rule induced by preference relation $\widehat{P}_c$.

Then run COP with students preferences $(P_s)_{s\in S}$ and constructed college choice rules $(\widehat{\mathcal{C}}_c)_{c\in C}$.

When colleges are restrictive, we propose a mechanism, whose outcome is found using the following procedure for any given market P:

Cumulative Offer Process without Immediately Eligible Transfers (COP-no-*I*):

Construct a preference relation $\widehat{P}_s$ from P_s for each student s as follows: Move contract $(s, \omega(s), W)$ and all term-I contracts with any college different from $\omega(s)$ below $x_\emptyset$, while maintaining the relative order of the other contracts.

Construct a preference relation $\widehat{P}_c$ from P_c for each college c as follows: Move contract (s, c, I) for all $s \in S_c$ above all other contracts while maintaining the relative order of the other contracts. Let $\widehat{\mathcal{C}}_c$ be the choice rule induced by preference relation $\widehat{P}_c$.

Then run COP with constructed student preferences $(\widehat{P}_s)_{s \in S}$ and constructed colleges choice rules $(\widehat{\mathcal{C}}_c)_{c \in C}$.

Next, we show that student-strategy-proof and stable mechanisms exist when colleges are either unrestrictive or restrictive, and the above two mechanisms are two of these mechanisms, respectively.

Proposition 1 *When colleges are unrestrictive, the COP-Unr mechanism is stable and strategy-proof, and when colleges are restrictive, the COP-no-I mechanism is stable and strategy-proof.*[19]

When colleges are unrestrictive, COP-Unr is also constrained efficient. We formally state this result in the following proposition.

Proposition 2 *The COP-Unr mechanism is constrained efficient but not undominated stable (and, therefore not Pareto efficient).*

So far, we have shown that, when colleges are unrestrictive, COP-Unr is stable, constrained efficient, and strategy-proof. Moreover, its outcome may include transfers with the term I. In fact, if colleges prefer the contract of any given student with term I over the one with term W, then all transfers are with term I. Hence, we can conclude that when colleges are unrestrictive, there exists a mechanism that satisfies this market's desired features. On the other hand, COP-no-I does not possess good efficiency properties when colleges are restrictive, as we analyze in the next subsection.

Nevertheless, these two mechanisms also play an important role in our empirical study in Section 6. When immediately eligible transfers are not allowed in a market, COP-no-I is used to emulate the market mechanism. In our empirical case study, this regime corresponds to the period before the 2021 reform. On the other hand, the reform removes the rights of colleges to contest the immediate eligibility of their transfer students. Therefore, COP-Unr is used to emulate the post-2020 market.

[19] The proposition holds even only Assumption 1 (i) and (ii) hold. We prove the proposition in this form. All proofs are delegated to Appendix A.

3.2 Efficiency and Stability with Restrictive Colleges

Although COP-no-I is a stable and strategy-proof mechanism when colleges are restrictive, it does not allow students to transfer with term I, which is undesirable for our environment.[20] Moreover, COP-no-I is not even undominated-stable. It is natural to ask whether we can do better in terms of efficiency when colleges are restrictive. To answer this question, we first analyze the properties of stable matchings with restrictive colleges.

The following example illustrates an important feature of stable matchings when colleges are restrictive.

Example 2 *We consider the same setting as in Example 1: There are three colleges, $C = \{a, b, c\}$, and two students, $S = \{s, s'\}$. Let $S_a = \{s\}$, $S_b = \{s'\}$, $S_c = \emptyset$. Colleges are restrictive, so that $C^R = C$. Each college has one seat. The preferences of students and colleges over contracts are given as:*

P_s	$P_{s'}$	P_a	P_b	P_c
(s, b, I)	(s', a, I)	(s', a, I)	(s, b, I)	$\vdots$
(s, b, W)	(s', a, W)	(s, a, I)	(s', b, I)	
(s, a, I)	(s', b, I)	(s', a, W)	(s, b, W)	
(s, a, W)	(s', b, W)	(s, a, W)	(s', b, W)	
$\vdots$	$\vdots$	$x_\emptyset$	$x_\emptyset$	

Stable matchings are given as follows:

$$\mu^1 = \{(s, a, I),\ (s', b, I)\},$$
$$\mu^2 = \{(s, b, W),\ (s', a, W)\},$$
$$\mu^3 = \{(s, b, I),\ (s', a, I)\}.$$

We can Pareto rank these three matchings: μ^3 Pareto dominates μ^2 and μ^1, while μ^2 Pareto dominates μ^1. In fact, μ^3 is the unique stable, Pareto-efficient, and therefore, undominated-stable matching. Moreover, only μ^1 and μ^3 are stable and constrained-efficient matchings. Notice that student-college pair (s, b) standard-blocks μ^1 and μ^2 with contract (s, b, I). However, that standard-block would lead to a matching that violates incontestability.

In the standard model of many-to-one matching with contracts, where choice rules satisfy substitutability and LAD, this Pareto rankability of stable matchings is not possible. There is a polarity of interest between colleges and students that results in a lattice structure for the set of stable matchings, where the partial lattice order is defined by increasing common preferences of students and decreasing common preferences of colleges (see Hatfield and Milgrom,

[20] This follows from the following observation: Under COP-no-I, for any market, each student s is matched with a contract weakly higher ranked than $(s, \omega(s), I)$ under $\widehat{P}_s$ (see the proof of Proposition 1) and all contracts (s, c', I) where $c' \neq \omega(s)$ are ranked below $(s, \omega(s), I)$ under $\widehat{P}_s$.

2005). Example 2 illustrates that this polarity is not observed in our setting when colleges are restrictive.

In contrast to the standard model of matching with contracts, Example 2 points out that stability does not imply (two-sided) Pareto efficiency. Moreover, the comparison between μ^2 and μ^3 shows that we can have a Pareto improvement for both colleges and students by only changing the terms of the contracts, i.e., μ^2 is not constrained efficient (and therefore not Pareto efficient). Although there exists a stable and constrained-efficient matching in Example 2, stability and constrained efficiency are not generally compatible properties in the transfer market when all colleges are restrictive.[21] The following theorem shows that in some markets, when colleges are restrictive, any stable matching can be Pareto-dominated by another uncontested matching that is partner-equivalent.

Theorem 1 *Suppose colleges are restrictive. There may not exist a stable matching that is constrained efficient, even under Assumption 1 and when $q_c \leq 1$ for all colleges but one.*

The following result is immediate:

Corollary 1 *Suppose colleges are restrictive. There does not exist a stable and constrained-efficient (and therefore Pareto efficient) mechanism.*

As a side remark, notice that in Example 2, colleges are restrictive and have the unit capacity, and at least one stable and constrained efficient matching exists. Moreover, the statement of Theorem 1 allows at least one college to have a capacity greater than 1, and its proof uses such an example. In the following section, we also show that this is not a coincidence, and indeed, Theorem 1 is tightly stated and proven: When colleges are restrictive and have unit capacity, there always exists a stable and constrained efficient matching.

3.3 Mechanism Design with Restrictive Colleges

In this section, we introduce a stable mechanism that allows transfers with contract I when colleges are restrictive. Our proposed mechanism utilizes the features of both COP and the Top Trading Cycles (TTC) mechanism. The key innovation in our mechanism is that it endogenously checks for incontestability, which equalizes the exports and imports with the term I for all colleges. We refer to our mechanism as **Uncontested Cumulative Offer Process** (**UCOP**). It is defined in four stages as follows:

Uncontested Cumulative Offer Process (UCOP):

Stage 1: Initialization

Quota Setting: For each college c, define two quotas q_c^I and q_c^A, where q_c^I is the maximum number of contracts that can be assigned to the college c with term I and q_c^A is the maxi-

[21]Recall that when colleges are unrestrictive, COP-Unr is stable, constrained efficient, and strategy-proof for students.

mum total number of contracts that can be assigned to college c. Set $q_c^I = |S_c|$ and $q_c^A = q_c$ for each college c.

Preference Update: For each college c, construct a preference order $\overline{P}_c$ from P_c by moving the contracts including its initial students with term I to the top of the list. Let $\overline{P}_S = P_S$ for all $s \in S$.

Auxiliary Choice Rule with an I-Specific Quota : For each college c, construct an auxiliary choice rule, $\overline{\mathcal{C}}_c$. Given a subset of contracts $X' \subseteq X_c$, the chosen set of contracts, denoted by $\overline{\mathcal{C}}_c(X')$, is determined through the following procedure:

Step 0: Set $X^1 := X'$, $Y^1 = \emptyset$, $q_c^{A,1} := q_c^A$ and $q_c^{I,1} := q_c^I$.

Step k: (k $\geq$ 1) Let $x_k := \max_{\overline{P}_c} X^k$.

If $x_k \neq x_\emptyset$:

1. Set $Y^{k+1} = Y^k \cup \{x_k\}$ and $q_c^{A,k+1} := q_c^{A,k} - 1$.
2. If $\mathrm{t}(x_k) = I$, then set $q_c^{I,k+1} := q_c^{I,k} - 1$. Otherwise, set $q_c^{I,k+1} := q_c^{I,k}$.
3. If $q_c^{I,k+1} = 0$, then set

$$X^{k+1} := X^k \setminus \left(X^k_{\mathrm{s}(x_k)} \cup X^k_I\right).$$

   Otherwise, set

$$X^{k+1} := X^k \setminus X^k_{\mathrm{s}(x_k)}.$$

If $x_k = x_\emptyset$: Set $Y^{k+1} := Y^k$, $X^{k+1} := X^k$, $q^{A,k+1} := q^{A,k}$, and $q^{I,k+1} := q^{I,k}$.

If $x_k = x_\emptyset$ or $q_c^{A,k+1} = 0$ or $X^{k+1} = \emptyset$, terminate the procedure by setting

$$\overline{\mathcal{C}}_c(X') := Y^{k+1}.$$

Otherwise, continue with Step k+1.

Stage 2: The COP Stage

Apply COP using student preference profile $\overline{P}_S$ and auxiliary college choice rule profile $(\overline{\mathcal{C}}_c)_{c \in C}$. Denote the obtained matching by μ.

Stage 3: Stability Check

Two cases are possible regarding μ:

Case 1. *μ is contested:* Contesting colleges are those with more exports with term I than imports with term I. Denote the set of contesting colleges by $\widetilde{C}$:

$$\widetilde{C} = \left\{ c \in C \;:\; \left|\mu_c^e \cap \mu_I\right| > \left|\mu_c^m \cap \mu_I\right| \right\}.$$

Denote the set of students exported by a college in $\widetilde{C}$ with term I under μ by $\widetilde{S}$:

$$\widetilde{S} = \bigcup_{c \in \widetilde{C}} \mathrm{s}(\mu_c^e \cap \mu_I).$$

Choose a student $s \in \widetilde{S}$. Let $\widetilde{c} = \mathrm{c}(\mu_s)$, and $\widetilde{x}$ be the lowest ranked contract in $\mu_{\widetilde{c}} \cap \mu_I \cap \mu^e_{\omega(s)}$ according to $\overline{P}_{\widetilde{c}}$. Let $s' = \mathrm{s}(\widetilde{x})$. Update the preference relation $\overline{P}_{s'}$ of s' by moving $(s', \widetilde{c}, I)$ below $x_\emptyset$, and go back to Stage 2 using the updated student preferences $\overline{P}_S$.

Case 2. *μ is uncontested:* Check whether there exists a student-college pair that blocks μ. If such a blocking pair exists, choose a blocking pair (s, c) and let t be their blocking term.[22] Let $\widetilde{x}$ be the lowest ranked contract in $\mu_c \setminus \mathcal{C}_c\big(\mu_c \cup \{(s, c, t)\}\big)$ according to $\overline{P}_c$.[23] Let $s' = \mathrm{s}(\widetilde{x})$. Update preference relation of s' by moving $\widetilde{x}$ below $x_\emptyset$. Go back to Stage 2 using the updated student preferences.

If such a blocking pair does not exist, continue to Stage 4.

Stage 4: Welfare Improvement

We consider welfare-improving cycles under matching μ. These welfare improving cycles do not change the assigned colleges of the students**.**

Given matching μ, construct the following directed graph among students and colleges.

- Each college c points to each student $s \in S_c$ such that $\mathrm{t}(\mu_s) = W$.
- Consider a student s with $\mathrm{t}(\mu_s) = W$ and $\mathrm{c}(\mu_s) \neq \omega(s)$.

  If there do not exist a student $\overline{s} \in S_{\mathrm{c}(\mu_s)}$ such that

  (i) $\mathrm{t}(\mu_{\overline{s}}) = I$,

  (ii) $(s, \mathrm{c}(\mu_{\overline{s}}), I)\ P_s\ (s, \mathrm{c}(\mu_s), I)$, and

  (iii) $(s, \mathrm{c}(\mu_{\overline{s}}), I)\ P_{\mathrm{c}(\mu_{\overline{s}})}\ (\overline{s}, \mathrm{c}(\mu_{\overline{s}}), I)$,

  and a student $s' \in S$ such that

  (i) $(s', \mathrm{c}(\mu_s), I)\ P_{\mathrm{c}(\mu_s)}\ (s, \mathrm{c}(\mu_s), I)$,

  (ii) $(s', \mathrm{c}(\mu_s), I)\ P_{s'}\ \mu_{s'}$, and

  (iii) either $[\mathrm{t}(\mu_{s'}) = W$ and $s' \in S_{\omega(s)}]$ or $\mu_{s'} = (s', \omega(s), I)$,

  then s points to $\mathrm{c}(\mu_s)$. Otherwise, s does not point to any college.

  Repeat this for each student.

If there exists a cycle in this graph, then in each cycle, each student s in the cycle is matched with $(s, \mathrm{c}(\mu_s), I)$ instead of $\mu_s = (s, \mathrm{c}(\mu_s), W)$. Let μ now denote the updated

[22] An external tie-breaker priority order can be used to choose a member of $\widetilde{S}$ in Case 1 and a blocking pair in Case 2.

[23] In the proof of Theorem 2, we show that $\mu_c \setminus \mathcal{C}_c\big(\mu_c \cup \{(s, c, t)\}\big)$ is singleton.

matching and go back to Stage 4.

If no cycle exists, we end the procedure with outcome matching μ.

We explain how UCOP works. Given Observation 1, in Stage 1, UCOP limits the number of contracts with term I that a college can be matched with under a stable matching. Then we define an auxiliary choice rule for each college that will be used in Stage 2. This auxiliary choice rule builds on a college's real choice rule to ensure that the number of tentatively assigned students with term-I contracts never exceeds the number of initial students at each college when COP is run.

In Stage 2, we run COP using the auxiliary choice rules and updated preferences of the students.

UCOP's intermediate outcome at the end of Stage 2 may violate stability through a contesting college or a blocking pair. If that is the case, then Stage 3 first checks whether the outcome obtained in Stage 2 is contested. If it is, we update the preference relation of an exported student of a contesting college, who is determined by the tie-breaker priority order, by moving the contract that causes a violation of incontestability below $x_{\emptyset}$. If Stage 2's outcome is uncontested but includes a blocking pair, we update the preference relation of a student by moving the contract through which this student forms a blocking pair below $x_{\emptyset}$. We reapply Stages 2 and 3 until we get a stable, and therefore uncontested, outcome.

In Stage 4, finally, we improve the welfare of both sides of the market while respecting stability using improvement cycles in Stage 4 without changing the assigned colleges of the students. This stage features a tractable check condition, which guarantees not to violate stability when the contracts are swapped to term I from term W for some matches. Other less tractable exhaustive search conditions can be embedded in here.

We illustrate UCOP's execution in Example 6 in Appendix B.

The following theorem shows that UCOP selects a stable matching, even though it can facilitate term-I transfers (see the example mentioned above).

Theorem 2 *Suppose colleges are restrictive. The UCOP mechanism is stable.*

The following result is a direct corollary of Theorems 1 and 2.

Corollary 2 *When colleges are restrictive, the UCOP mechanism is not constrained efficient (and therefore, not Pareto efficient).*

Although UCOP is not constrained efficient, whenever every college has a unit capacity, UCOP selects a constrained efficient matching.

Proposition 3 *Suppose colleges are restrictive. If $q_c \leq 1$ for all $c \in C$, then the UCOP mechanism is stable and constrained-efficient.*[24]

[24]Since we require $|S_c| \geq q_c$ for all $c \in C$, $q_c = 0$ implies $S_c = \emptyset$.

Theorem 1 and Proposition 3 show that when colleges are restrictive, there does not exist another stable mechanism that selects constrained efficient matching in the domain of the markets, particularly quota profiles, where UCOP's outcome is constrained inefficient.

Note that in the standard model of many-to-one matching with contracts, where choice rules satisfy substitutability and LAD, it is well known that there exists a stable, efficient, and strategy-proof mechanism for students (Hatfield and Milgrom, 2005). However, this result does not hold in our environment, even if we restrict all colleges to unit capacity and consider constrained efficiency.

Theorem 3 *Suppose colleges are restrictive. There is no stable, constrained-efficient, and student-strategy-proof mechanism, even when all colleges have unit capacity.*

Theorems 1 and 3 give us the limitations of stable mechanisms when colleges are restrictive: they are neither strategy-proof nor constrained efficient. Since UCOP is stable with restrictive colleges, it is neither strategy-proof nor constrained efficient. Then, one can wonder whether another stable mechanism exists that performs better than UCOP in all dimensions. The answer is no, even if we consider the larger class of uncontested mechanisms. The following two results show that any possible improvement over UCOP in one dimension is costly in another dimension.

Proposition 4 *Suppose colleges are restrictive. There does not exist an uncontested and strategy-proof mechanism that Pareto dominates the UCOP mechanism.*[25]

Before stating our next result, we define a comparison notion that can be used to compare any two mechanisms based on their vulnerability to manipulation.[26] A mechanism ϕ is **less manipulable** than another mechanism ψ, if whenever a student can gain from misreporting his preferences under ψ, then some students can also gain from misreporting under ϕ and there exists at least one market such that no student can gain from misreporting under ϕ but at least one student can gain from misreporting under ψ. We also say that a mechanism ϕ **Pareto improves** another mechanism ψ if for every market P, $\phi(P)$ weakly Pareto dominates $\psi(P)$ and whenever $\psi(P)$ is Pareto inefficient, $\phi(P)$ Pareto dominates $\psi(P)$.

Proposition 5 *Suppose colleges are restrictive. Any mechanism that Pareto improves UCOP cannot be less manipulable than the UCOP mechanism.*

3.4 Transfers in Markets without Initially Vacant Positions

When colleges are unrestrictive, earlier we introduced a mechanism, COP-Unr, which is strategy-proof for students, stable, constrained efficient, and allows transfers with term I. However, constrained efficiency and stability are generally incompatible when all colleges are

[25] A mechanism ϕ Pareto dominates another mechanism ψ if for every market P, $\phi(P)$ weakly Pareto dominates $\psi(P)$ and for some market P' $\phi(P')$ Pareto dominates $\psi(P')$.

[26] This comparison notion was first introduced by Pathak and Sönmez (2008).

restrictive. When we prove this incompatibility result (Theorem 1), we use a market where one of the colleges starts with a vacant position. In this subsection, we obtain stronger results by considering markets without initially vacant positions (i.e., $q_c = |S_c|$ for every college $c \in C$). These markets are relevant when the market starts in a situation that meets the balance condition imposed by the policymaker. We have an additional mild assumption on colleges' preferences.

Assumption 2 *For each college $c \in C$ and students $s, s' \in S$,*

$$(s, c, I)\ P_c\ (s', c, I) \quad \Longrightarrow \quad (s, c, I)\ P_c\ (s', c, W).$$

Under Assumptions 1 and 2 in a market without initially vacant positions but with restrictive colleges, the execution of UCOP simplifies because Stages 3 and 4 become redundant. That is, the outcome of COP in Stage 2 is always stable, and all transfers have a term-I contract. Moreover, in Stage 1, since $|S_c| = q_c$ for every $c \in C$, we do not need to define a specific quota for term-I contracts. Hence, in the computation of COP in Stage 2, for every college c, we can use the choice rule $\mathcal{C}_c$ induced by updated preference relation $\overline{P}_c$. This gives us a simplified version of the UCOP mechanism, which is composed of Stages 1 and 2.[27]

First notice that UCOP and COP-Unr become equivalent in this case. Although UCOP was not a relevant mechanism when colleges were unrestrictive in general, this equivalence implies that UCOP can also be used with unrestrictive colleges when there are no vacant positions under Assumptions 1 and 2. Moreover in this case, COP-Unr is stable, constrained efficient, and student-strategy-proof (Propositions 1 and 2). Next we show that, under Assumptions 1 and 2, UCOP inherits these properties and all transfers have a term-I contract even when colleges are restrictive.

Theorem 4 *Suppose colleges are restrictive, $q_c = |S_c|$ for all $c \in C$, and Asummption 2 holds (in addition to Assumption 1). Then, the UCOP mechanism is stable, constrained efficient, and strategy-proof for students. Moreover, in any market, any transfer in its outcome matchings has a term-I contract.*

Theorem 4 implies that UCOP is outcome equivalent to a modification of the deferred acceptance algorithm algorithm in two-sided matching markets such that students only consider term-I contracts acceptable, and the colleges have responsive preferences over the contracts. Hence, its outcome is expected to be Pareto efficient and, therefore, undominated stable. However, since we construct colleges' choice rules by considering the updated rankings over the contracts, UCOP's outcome is not guaranteed to be undominated stable. In fact, when $q_c = |S_c|$ for every $c \in C$ and $q_{c'} > 1$ for at least one college c', a strategy-proof mechanism cannot select an undominated-stable matching. We illustrate this incompatibility in the following example.

[27] All these observations are shown in the proof of Theorem 4.

Example 3 *There are three colleges* $C = \{a, b, c\}$ *with capacities* $q_a = 2$, $q_b = 1$, *and* $q_c = 1$. *Let* $S_a = \{s, s'\}$, $S_b = \{s''\}$, *and* $S_c = \{\widetilde{s}\}$. *Preferences satisfy Assumptions 1 and 2 and are given as:*

P_s	$P_{s'}$	$P_{s''}$	$P_{\widetilde{s}}$	P_a	P_b	P_c
(s,b,I)	(s',c,I)	(s'',a,I)	$(\widetilde{s},b,I)$	(s'',a,I)	(s',b,I)	(s,c,I)
(s,a,I)	(s',a,I)	(s'',b,I)	$(\widetilde{s},c,I)$	$(\widetilde{s},a,I)$	(s,b,I)	(s',c,I)
$\vdots$	$\vdots$	$\vdots$	$\vdots$	(s,a,I)	$(\widetilde{s},b,I)$	$(\widetilde{s},c,I)$
				(s',a,I)	(s'',b,I)	$\vdots$
				$\vdots$	$\vdots$	

In any stable matching, all colleges fill their capacity and students are assigned to contracts weakly better than their initial contracts. Therefore, all contracts are with term I in any stable matching. As a result, the sets of stable matchings (and therefore undominated stable matchings) under the cases of all restrictive colleges and all unrestrictive colleges are the same. In any undominated-stable matching, s'' *is matched with* (s'', a, I). *This is due to the following observation: In a stable matching* ν, *if* s'' *is not matched with* (s'', a, I), *then he needs to be matched with* (s'', b, I) *by individual rationality. Then, this implies that* $\nu_s = (s, a, I)$ *and* $\nu_{\widetilde{s}} = (\widetilde{s}, c, I)$ *by individual rationality. These imply* $\nu_{s'} = (s', a, I)$. *Thus,*

$$\nu = \{(s,a,I),\ (s',a,I),\ (s',b,I),\ (\widetilde{s},c,I)\}.$$

However, matching ν *is Pareto dominated by two other stable matchings:*

$$\nu' = \{(s,b,I),\ (s',a,I),\ (s'',a,I),\ (\widetilde{s},c,I)\} \quad \text{and} \quad \nu'' = \{(s,a,I),\ (s',c,I),\ (s'',a,I),\ (\widetilde{s},b,I)\}.$$

In particular, ν' *and* ν'' *are the only undominated-stable matchings in this market, and in both matchings,* s'' *is matched with contract* (s'', a, I).

Consider an undominated-stable mechanism selecting ν' *in market P. Suppose* s' *reports*

$$P'_{s'}: \quad (s',c,I)\ P'_{s'}\ (s',b,I)\ P'_{s'}\ (s',a,I)\ \ldots$$

and all other students report their true preferences. Then, under market $(P'_{s'}, P_{-s'})$, ν' *is no longer stable. In particular,* (s', b) *blocks* ν' *with term I. Under* $(P'_{s'}, P_{-s'})$, *as explained above for market P, under any undominated-stable matching,* s'' *is matched with* (s'', a, I). *In particular,* ν'' *is the unique undominated-stable matching under market* $(P'_{s'}, P_{-s'})$. *Hence, any undominated-stable mechanism selecting* ν' *under market P can be manipulated by* s'.

Consider an undominated-stable mechanism selecting ν'' *in market P. Suppose s reports*

$$P'_s: \quad (s,b,I)\ P'_s\ (s,c,I)\ P'_s\ (s,a,I)\ldots$$

and all other students report their true preferences. Then, under this market, i.e., (P'_s, P_{-s}), ν'' *is no* *longer stable. In particular, (s, c) blocks ν'' with term I. Under (P'_s, P_{-s}), as explained above for market P, under any undominated-stable matching, s'' is matched with (s'', a, I). In particular, ν' is the unique undominated-stable matching under market (P'_s, P_{-s}). Hence, any undominated-stable mechanism selecting ν'' under market P can be manipulated by s.*

Given the incompatibility illustrated in Example 3, in which one college has two available positions, we focus on markets where $q_c = |S_c| = 1$ for every $c \in C$. When we have $q_c = |S_c| = 1$ for every $c \in C$ with restrictive colleges, then the following mechanism, **Top Trading Cycles with Contracts (TTC-wC)**, is Pareto efficient, stable, and strategy-proof under Assumption 1.[28]

Top Trading Cycles with Contracts (TTC-wC):

Step 1: Each college c points to the unique student $s \in S_c$. Each student s points to college $\mathrm{c}(x^s)$ such that x^s is the most preferred contract of student s that is acceptable by the corresponding college. Then by Assumption 1 and finite sets of students and colleges, there exists a cycle. We execute the cycles by assigning every student s in that cycle to the college $\mathrm{c}(x^s)$ with term $\mathrm{t}(x^s)$. Each assigned student and college is removed.[29]

In general,

Step k: (k > 1) Each remaining college c points to student $s \in S_c$. Each remaining student s points to college $\mathrm{c}(x^s)$ such that x^s is the most preferred contract of student s that is acceptable by the corresponding college among the remaining colleges. Then by Assumption 1 and finite sets of students and colleges, there exists a cycle. We execute the cycles by assigning every student s in that cycle to the college $\mathrm{c}(x^s)$ with term $\mathrm{t}(x^s)$. Each assigned student and college is removed.

The procedure terminates when all students and colleges are removed.

Proposition 6 *Suppose colleges are restrictive, $q_c = |S_c| = 1$ for all $c \in C$. The TTC-wC mechanism is stable, Pareto efficient, and strategy-proof for students.*

Observe that since the definition of TTC-wC includes contracts, Pareto efficiency, and strategy-proofness properties are not directly implied by the previous results on the TTC mechanism.

4 Overview of Colleges Transfers and the NCAA

To empirically test the performance of the Uncontested Cumulative Offer Process (UCOP), we use data from a real-world transfer market. Our empirical focus is on the transfer market for student-athletes in US colleges because athletic settings have rich data that allow the researcher to precisely estimate demand. The policymaker of interest in this setting is the NCAA,

[28] When colleges are unrestrictive, the TTC mechanism may fail to be stable.

[29] Notice that a student s is removed if and only if $\omega(s)$ is removed.

which administers the current student-athlete transfer process and sets its rules. Colleges annually recruit high school athletes for their athletic teams, where American football and men's basketball are the most prominent sports in terms of revenue and viewership. High school athletes visit colleges and are visited by officials of college teams. Subsequently, the athlete chooses which college to attend. Some years after enrolling, some student-athletes would like to transfer to another college for various reasons. After transferring, prior to 2021, in most cases, a player needed to wait one year to be eligible to play for their new college. If the NCAA approved a player's appeal with hardship, he had immediate eligibility to play. The stated purpose of the NCAA's *sit-out rule* is to help players adapt to the new college.[30]

As of 2021, the NCAA changed its transfer rules to make it easier for players to be immediately eligible. As of 2023, the NCAA again implemented changes that have obscured the de facto set of transfer rules and left a great deal of ambiguity.[31] During each of these policy regimes, the NCAA transfer process has received substantial criticism. For a specific example, a USA Today article discussed the 2021 NCAA transfer rules changes, then provided a comparison with the state of transfers circa 2022. For efforts that instigated the 2021 policy changes, Tom Mars was described as

> "a disruptor to the NCAA's outdated and often draconian restrictions on player transfers." (Wolken, 2022)

Then when discussing changes to the NCAA transfer process as of 2022, Mars stated:

> "This is a really, really bad idea. That was never my intention to help go from one end of the spectrum to the complete other end of the spectrum... What I think is coming down the pike [is] utter chaos." (Wolken, 2022)[32]

Our goal is to propose a solution in settings such as these where there is a need for balance

[30] The NCAA's website said that "requiring student-athletes to sit out of competition for a year after transferring encourages them to make decisions motivated by academics as well as athletics. Most student-athletes who are not eligible to compete immediately benefit from a year to adjust to their new school and focus on their classes."

[31] A widely discussed example in 2023 was the NCAA ruling on football player Tez Walker's eligibility after Walker did not receive a waiver for immediate eligibility. See https://www.cbssports.com/college-football/news/ncaa-responds-to-unc-over-criticism-from-tez-walker-eligibility-ruling-acknowledges-receiving-threats.

[32] See https://www.usatoday.com/story/sports/college/columnist/dan-wolken/2022/07/29/college-athlete-lawyer-tom-mars-ncaa-transfer-system-chaos/10185541002.

rather than oscillating between extreme regimes.[33]

Our specific setting of interest is NCAA Division I men's basketball, where Division I is the NCAA's top level, including the most prominent colleges and players. According to statistics from the NCAA, there were 5,485 student-athletes on men's basketball Division I rosters in 2022, where 1,649 students declared an intention to transfer and 1,123 students in fact transferred.[34] These statistics show that the one-year transfer rate in men's basketball was around 20% in 2022, and over the course of four years of college, students have several years to transfer. Why is there such a large volume of transfers among student-athletes? First, note that there is a high transfer rate for all US college students of 37.96% (Shapiro et al., 2018). Explanations for the overall transfer rate focus on ex-ante planned transfers (e.g., enrolling in a two-year college with plans to transfer to a four-year college, perhaps to save money at a less expensive two-year college) or unplanned transfers (e.g., in response to academic failures or failures to adjust); see Jenkins and Fink (2015). Turning to transfers among student-athletes, Richards, Holden, Pugh, et al. (2016) provide a detailed discussion, but here we overview the common rationales for student-athletes to transfer to a new college. When transferring, student-athletes commonly express concern about insufficient playing time,[35] deteriorating relationships with the coach or other staff,[36] the departure or firing of the coach who recruited them,[37] or changes

[33]There are numerous anecdotes of coaches, colleges, players, and commentators who are unhappy with transfer rules. Consider the transfer rules pre-2021. One example is from a Washington Post article entitled "NCAA transfer rules are pointless. And also randomly enforced." See https://www.washingtonpost.com/sports/colleges/ncaa-transfer-rules-are-pointless-and-also-randomly-enforced-ask-brock-hoffman/2019/08/29/5ffb33a4-ca94-11e9-a1fe-ca46e8d573c0_story.html. There are numerous other examples of major criticism of the current transfer process. According to the USA Today: "While many athletes such as high-profile [players] ... have been granted waivers for reasons that publicly appear to be ambiguous" (https://www.usatoday.com/story/sports/college/columnist/dan-wolken/2019/06/26/ncaas-changes-transfer-guidelines-limit-immediate-eligibility/1569260001). Further, a writer for SB Nation said that the NCAA was "denying waivers for players with family medical issues but approving them for [quarterbacks] stuck on the bench" (https://www.sbnation.com/2019/4/24/18514525/ncaa-transfer-waivers). Finally, see a New York Times editorial entitled "How the NCAA Cheats Student Athletes" (https://www.nytimes.com/2017/10/03/opinion/how-the-ncaa-cheats-student-athletes.html).

[34]See https://www.ncaa.org/news/2023/2/21/media-center-2022-transfer-trends-released-for-divisions-i-and-ii.aspx. The number of transfers in earlier years is as follows: 689 actual transfers in 2017; 704 actual transfers in 2018; and 1,066 intended and 694 actual transfers in 2019 (https://ncaaorg.s3.amazonaws.com/research/transfers/Jan2020RESDestD1MBBTransfers.pdf).

[35]For example, Marcus Lee transferred from Kentucky to California in 2016 for this reason. See https://www.sbnation.com/college-basketball/2016/5/26/11781488/marcus-lee-transfer-kentucky-recruiting-bam-adebayo-wenyen-gabriel.

[36]For example, Derryck Thornton transferred from Duke to Southern California in 2016 for this reason. See https://www.sportingnews.com/us/ncaa-basketball/news/derryck-thornton-duke-transfer-mark-edwards-uncle/13nrh9atrjez0119lu0a3cqtiw.

[37]For example, Danuel House transferred from Houston to Texas A&M in 2014 following a coaching change. See https://www.sportingnews.com/us/ncaa-basketball/news/danuel-house-texas-am-eligibility-houston-sampson-ncaa-waiver-transfer-cleared-to-play/1kosxdqi038qn1n3wdauxw6iu5.

in their attitude about the college itself, such as its location.[38]

5 Data and Identification

5.1 Data

Our empirical analysis uses the following data to analyze matching within the NCAA transfer market:

(1) data from the initial recruitment period, when colleges recruit high school athletes;

(2) data from the transfer period, when students at a given college transfer to another college; and

(3) data on colleges in terms of statistics that evaluate college-level performance.

We use information from the initial recruitment period to estimate students' and colleges' preferences.[39] The basic idea in estimating students' preferences is that a student's decisions during his initial recruitment period reveal information about his preferences. For students, we observe the set of colleges that made them an offer and which college's offer they accepted. This is the college where they initially enrolled. We estimate students' preferences over college *attributes* rather than colleges from the initial recruitment period. We then use those preferences over attributes to estimate the preferences over colleges in future years because the transfers happen some years after the initial recruitment. Consider a student whom two colleges recruit; assume that colleges differ in performance (e.g., wins) and style (e.g., offensive versus defensive focus). Say, college A won more games but has a defensively focused style, while college B won fewer games but has an offensively focused style. If the student received offers from both colleges, then choosing to enroll at college B is evidence that he prefers playing in an offensively focused style, ceteris paribus. If he transfers one year later, we infer his preferences over colleges in the transfer year based on colleges' performance and style as of the transfer year. The basic idea in estimating colleges' preferences is similar. For colleges, we observe the students to whom they made an offer and which of these students accepted their offers.

We collect data during the period in which the NCAA transfer market was in its fully restrictive regime. The recruiting data cover the period from 2010 to 2017, and the transfer data

[38] For example, Devonta Pollard transferred from Alabama to East Mississippi Community College in 2013 to return to his hometown, following legal problems involving his mother. See http://www.espn.com/espn/feature/story/_/id/11783790/can-former-alabama-top-recruit-devonta-pollard-start-houston.

[39] It is important to emphasize that policymakers (e.g., the NCAA) do not need to estimate preferences to implement our proposed solutions. In practice, students and colleges would submit information to a centralized match; our proposed solutions would then take these submitted preferences as inputs. We estimate preferences to show the possible improvements with the tools from market design. We find substantial improvements over the current ad hoc process that has been heavily criticized.

from 2012 to 2018.[40] The starting point for the data was the universe of students available in the 247 Sports recruiting data each year. The 247 Sports recruiting data covers the US's top high school basketball players. We collected data on around 250 students in a typical year in their data, and we found that students ranked lower than that threshold tend to have missing information. Thus, students who are not considered one of the top 250 recruits/year are not included in the recruiting data. The colleges included in our analysis are those colleges that made an offer to one of the top roughly 250 recruits, as determined by 247 Sports. As a result, these colleges will represent most NCAA Division I colleges, and the excluded colleges will be smaller colleges in Division I or colleges in Division II.

5.2 Identification

Given these data, the econometric model is identified following standard nonparametric identification arguments in discrete choice models (e.g., Matzkin, 1993). Unlike the earlier theoretical model of the transfer market, this discussion is based on the initial recruitment period because this period provides the data for our preference estimation. As before, C is the set of colleges, S is the set of students, P_c are colleges' preferences over students, and P_s are students' preferences over colleges. Students' preferences are a function of observable determinants (Z_s) and unobservable determinants (ε_s).[41] Colleges' preferences are a function of observable determinants (Z_c) and unobservable determinants (ε_c). The covariates in Z_s affect students' preferences as a function of a parameter vector β_s to be estimated, and similarly for (Z_c, β_c).

We make the following assumptions:

(i) conditional on Z_s and Z_c, ε_s and ε_c are independent; and

(ii) conditional on Z_s and Z_c, the draw from the distribution of ε_s for student s is independent of the set of colleges that make an offer to student s, for all $s \in S$.

See Fack, Grenet, and He (2019) for details as applied to matching markets and Matzkin (1993) for a discussion in a more general setting.

Further, for identification, we assume stability in the matching. For students' preferences, the variation in the data that identifies preferences is the observable characteristics of colleges

[40] Data from the recruitment period are from 247 Sports (http://www.247sports.com). Students' preferences depend on the college's location relative to the student's hometown, the college's performance, and the style of the college's play. Colleges' preferences depend on the student's hometown relative to the college's location, the student's quality of play in high school, and other student characteristics. In terms of high school performance, students are evaluated by experts known as scouts, which results in a scouting grade that cardinally summarizes their quality of play. College-level data and student-level data on playing time are from Sports Reference (https://www.sports-reference.com). Each student's hometown's distance to each college is calculated using a publicly available geocoder (https://www.geocod.io). College-level data about NBA draft picks are from ESPN (https://www.espn.com/nba/draft/rounds). Data from the transfer period are from Verbal Commits (http://www.verbalcommits.com).

[41] We use the notation Z_s for the covariates affecting students' preferences. Still, these covariates may vary across students and colleges. For example, one element of Z_s is the distance from the student's hometown to the college. The same slight abuse of notation should be noted for Z_c.

that predict which offer the student will accept, conditional on the set of offers he received.[42] For colleges' preferences, the variation in the data that identifies preferences is the observable characteristics of students that predict which students a college will make an offer to, conditional on the universe of students to whom they could make an offer.[43]

The econometric model to estimate preferences is in the spirit of Fack, Grenet, and He (2019), where our implementation uses McFadden's generalized conditional logit model. This model is equivalent to the rank-ordered logistic regression model when the data have information only on the favorite/chosen alternative, rather than a full rank-ordered list of preferences. In our setting, we know which college a student chose but do not know the preference order of the unchosen colleges. Likewise, for colleges' preferences, we know which offers a college made but do not have any information about their preference rankings. This results in McFadden's generalized conditional logit model. For students' preferences, we estimate preferences over college attributes via the model's estimates of attributes' point estimates. For colleges' preferences, we estimate preferences over students via the model's estimates of alternative-specific utility shifters.

For students' preferences, we know a student's set of offers and which college they chose. We assume stability in the matching and use Fack, Grenet, and He (2019)'s stability-based estimator, which is equivalent to McFadden's generalized conditional logit model. For students, the behavioral requirement implied by an assumption of stability is as follows: students must choose their most preferred college from among those that offered them admission.

For colleges' preferences, we know which students each college made offers to (out of the universe of students). For colleges, the behavioral requirement implied by an assumption of stability is as follows: each college must prefer every student they made an offer to over every student who (i) did not receive an offer and (ii) would have accepted if offered.[44] For colleges, unlike for students, we do not observe choice sets and instead know only the universe of students to whom they could make an offer.

As such, we construct a college's choice sets for each offer made by the college, where for college c, the choice set includes the student who received that offer from c along with other students who did not receive an offer from c yet who preferred c to the school whose offer

[42]The covariates used are whether the college and student are from the same state, the distance from his hometown to the college, measures of the college's winning success, and measures of the college's offensive and defensive style of play. Winning success is captured over the previous five seasons in terms of times ending the season among the Associated Press top 25 teams, number of NCAA Tournament appearances, average winning percentage, average strength of schedule, and the number of players drafted into the National Basketball Association (NBA). Shooting efficiency and three-point field-goal percentage measure offensive style of play. The rates of steals and blocks measure the defensive style of play.

[43]The covariates used are the student's scouting grade, in-state college, distance from hometown, and the student's height and weight.

[44]For example, suppose a college made an offer to student A but not to students B or C. Student B would have rejected the offer, but student C would have accepted. The behavioral requirement implied by stability requires that the college must prefer A to C with no restriction applied to the college's preferences with respect to B.

they accepted. More explanation is useful for two parts of this construction. First, the choice set conditions on the student's playing position (center, forward, or guard): the choice set for an offer to a center includes only other centers. Second, the choice set includes only other same-position students who would have accepted c's offer, which we identify via estimated students' preferences over colleges.[45] Once these choice sets are constructed, the behavioral requirement implied by an assumption of stability is as follows: colleges must make an offer to the most preferred student in the constructed choice set, where there is one choice set for each offer made by college c.

A final requirement for identification is a normalization to zero of one of the alternative-specific constants in Fack, Grenet, and He (2019)'s stability-based estimator. This is standard, and for students, we can arbitrarily normalize one of the college-specific constant terms in the usual way. For colleges however, we need to estimate well over 1,000 student-specific constant terms. For a given sample of students and colleges, Fack, Grenet, and He (2019)'s stability-based estimator is very demanding on the data in terms of precisely estimating the structural parameters along with the full set of alternative-specific constant terms. To address this issue, we normalize a set of student-specific constant terms to zero rather than only one. Specifically, we (i) construct choice sets for colleges as described above, (ii) tabulate the frequencies with which each student appears in the sample of all choice sets, (iii) mark those students who appear least frequently (i.e., in the first percentile of the frequency distribution), and (iv) estimate the model including student-specific constant terms for all remaining students (i.e., for the remaining 99% of the students).

5.3 Preference Estimation Results

Across the eight years of our recruiting data (2010-2017), there are 1,877 students in the 247 Sports recruiting data, and they received offers from 336 distinct colleges. Our estimation sample retains those colleges that made more than one offer over the entire eight-year period, which gives us an estimation sample of 324 colleges.[46] For context, consider 2016, where the year refers to students' high school graduation year and the initial year of a given basketball season (2016-2017 in this example). The college making the most offers was Kansas State with 40 offers, 3 of which were accepted; a more typical example is Villanova who made 5 offers, 2 of which were accepted. Overall in 2016, colleges made an average of 8.21 offers (median = 5) and 11.33% of those offers were accepted on average.

Using these data from the recruiting period, we estimate students' and colleges' preferences. Estimation results for students and colleges are in Tables 1 and 2, respectively. For students,

[45] That is, a student s can only be in the choice set of college c if s preferred c relative to the college whose offer s accepted (s's estimated utility of c is larger than estimated utility of c', where s is matched to c' in the assignment).

[46] There are roughly 350 Division I basketball programs, so our data cover the vast majority of Division I colleges. Examples of excluded colleges are Abilene Christian, Maine, and South Carolina Upstate. To emphasize, the exclusion is that a college had to make more than one offer over the entire eight-year sample period.

colleges in the same state are more highly preferred, but controlling for this, distance of the college from home is not a strong predictor of students' preferences. Omitting in-state leads to a strong negative effect of distance. The next set of covariates measures the college's winning success, though these are not strongly predictive. Students prefer colleges with more NBA Draft picks. The final set of covariates measures the style of play. For offensive play, the effects are estimated with a lot of noise, while for defensive play, students prefer defensive styles with higher rates of blocked shots.

For colleges' preferences, Table 2 shows that colleges prefer higher scouting grades. Colleges prefer students whose hometowns are closer; for in-state students, the effect is negative but as with students' preferences, in-state and distance are highly correlated. Finally, colleges prefer taller students, while the effect of weight is statistically insignificant.

These estimation results are not our main focus. As emphasized earlier, to implement our proposed mechanism, policymakers (e.g., the NCAA) would establish a centralized match in which students and colleges would report their preferences for where to transfer and whom to enroll, respectively. We conduct our estimation to demonstrate the improvements that are possible given that we do not have preference data. Now we turn to our main focus, which is the transfer process and how to measure improvements from a centralized transfers match.

6 Counterfactual Analysis

6.1 Counterfactual Mechanisms

Our counterfactuals present the results from three mechanisms. The **Uncontested Cumulative Offer Process (UCOP)** is our proposed mechanism to balance the objectives of agents on both sides of the market (here, colleges and students). We compare UCOP to two other mechanisms.

The **Before** mechanism is a simplified version of the NCAA transfer rules *before* the policy changes beginning with 2021. As described in Section 4, prior to 2021, it was difficult for students to transfer with immediate eligibility. We simulate the Before mechanism by running the *Cumulative Offer Process without Immediately Eligible Transfers* (*COP-no-I*) described in Section 3.1.

The **After** mechanism is a simplified version of the NCAA transfer rules *after* those policy changes. As described Section 4, rule changes in 2021 made it easier for students to transfer with immediate eligibility. We implement the After mechanism by running the transfer market with all colleges as unrestrictive colleges, specifically by running the *Cumulative Offer Process with Unrestrictive Colleges* (*COP-Unr*) mechanism described in Section 3.1.

Table 1: Results from Estimation of Students' Preferences

	(1)
In-State School=1	0.440
	(0.090)***
Distance from Hometown	-0.000
	(0.000)
Times AP Ranked, Last 5 Seasons	0.086
	(0.066)
Times in NCAA Tournament, Last 5 Seasons	-0.062
	(0.055)
Average Winning Percentage, Last 5 Seasons	-0.158
	(0.770)
Average Strength of Schedule, Last 5 Seasons	0.080
	(0.052)
# NBA Draft Picks, Last 5 Seasons	0.072
	(0.029)**
Average Shooting Efficiency	-3.053
	(2.307)
Average 3 Point Field Goal Percentage	0.924
	(1.953)
Average Rate of Steals	0.017
	(0.026)
Average Rate of Blocks	0.041
	(0.017)**
Observations	11,477

Notes: This table presents the regression results for the estimation of players' preferences. Regression coefficients are shown along with standard errors in parentheses; *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Table 2: Results from Estimation of Colleges' Preferences

	(1)
Scouting Grade	18.331
	(1.224)***
In-State Student=1	-0.118
	(0.057)**
Distance from Hometown	-0.001
	(0.000)***
Height	0.520
	(0.155)***
Weight	0.027
	(0.079)
Observations	368,157

Notes: This table presents the regression results for the estimation of colleges' preferences. Regression coefficients are shown along with standard errors in parentheses.

Each mechanism is run separately for each year between 2012 and 2016.[47] We start our analysis in 2012 because this is the first year in our transfers data. We end in 2016 because this year is well before any NCAA changes to the transfer process that we are studying in our counterfactuals. The years are labeled by the college-entry year. For example, 2012 is the year in which students enrolled in a college and played their freshman year in the 2012-2013 basketball season. Our counterfactual transfer market is run by following the cohort of freshman one year later and allowing them to potentially transfer after one year (after their freshman season). For example, the 2012 cohort of students potentially transfer in 2013 to potentially join a new college in their sophomore year. If a student transfers with contract term I, then they play for their new college in the 2013-2014 basketball season. If a student transfers with term W, then they wait out in the 2013-2014 season and can first play for their new college in the 2014-2015 season.

6.2 Counterfactual Preferences

We do not know how students' preferences change from initial enrollment to the time of their transfer decision, but it is intuitive for preference changes to play a role in transfers. Given the data estimated in the previous section, there are two elements we need to take a stand on to study our counterfactual matching markets. First, we model changes in students' preferences

[47] Our design can be generalized by considering multiple years of transfers. In particular, whenever a college A allows one of its outgoing students to play immediately at college B, then we say college B owes one immediate eligibility to college A. Then in the next year, if player b from B would like to play at college C and player c from college C would like to play at college A, then we can use A's credit at B to complete this transfer cycle.

over colleges at the time of potential transfers. Second, we calibrate the disutility of contract term W relative to term I (waiting out a year rather than being immediately eligible to play).

We model preference changes over time as follows: a student's utility at each college receives a positive shock that is i.i.d. from the continuous uniform distribution. We further impose a revealed preference assumption: if a student from initial college c transfers to college c' with term W, then we apply a negative shock to student i's utility at c with term I such that it is below student i's utility at c' with term W. With our approach using a continuous uniform distribution of positive shocks, we need to specify the upper bound (i.e., the largest shock possible) and consider the following upper bounds: 0.01, 0.05, 0.10, and 0.15. Second, we calibrate the term W disutility as a scalar and consider the following penalty scalars: 0.01, 0.05, 0.10, 0.15, 0.20, and 0.25. In what follows, the shocks to college utilities are called "preference shocks," and the contract term W disutility is called the "W penalty."

These required modeling choices are the upper bound for the preference shock distribution and the scalar for the W penalty. We set them as follows: repeatedly run counterfactual transfer markets with different values for these two modeling choices; measure the number of transfers in the counterfactual markets under the Before mechanism; compare the predicted number of Before transfers to the actual number of transfers in the transfer data; choose the parameters that best fit the data.[48] The data fitting identifies the parameters that most closely match the actual number of transfers in the transfer data for the years in our counterfactual analysis. Specifically, the parameters used in the main analyses are the parameters that provide the number of counterfactual transfers that most closely match the number of actual transfers year-by-year. The fit is measured by the sum of the squared differences (actual relative to predicted number of transfers), where the best fit is from the parameters that minimize the sum of the squared differences.

To understand the magnitudes of the parameters, first note that students' preferences from the initial recruitment period are predicted from the McFadden's generalized conditional logit estimates. These utility estimates are arbitrarily scaled from the linear predictions of the regression model, so we rescale them to $[0, 1]$. Relative to this utility range from 0 to 1, the parameters that best fit the actual transfer rates are a preference shock distribution of $U[0, 0.10]$ and a W penalty of 0.20. In summary, we shock student s's utility for college c with term I via a draw from a uniform distribution $U[0, 0.10]$ for all colleges c using student-college draws, then we subtract the W penalty scalar of 0.20 from the student's utility for all colleges c with term W.

The final element we need to take a stand on is how to specify a rule for setting colleges' quotas in the transfer market. In a centralized transfers match, colleges would report their preferences and their quotas (which students they want to enroll and the maximum number of such students). To run a counterfactual transfers match, we set colleges' quotas as follows:

[48] We use the counterfactual results from the Before mechanism to best fit the actual number of transfers because the Before mechanism is our approximation of the "mechanism" that was in place in those years, 2012-2016.

in the actual transfer data, observe the number of students who are from college c, and the number of students who, in fact, transferred to college c (imports) and students remaining at c; college c's quota is set equal to the maximum of the number of students initially assigned and finally assigned after the actual transfers.

In the following section, we compare mechanisms under the best fitting parameters. Relative to these modeling choices, we provide alternative results under different parameters after discussing the main results. In the counterfactual, all students potentially may transfer. The counterfactual is repeated 100 times to repeatedly draw from the shock distribution. In each simulation run, three mechanisms separately produce an assignment: Before, UCOP, and After. The inputs to the counterfactual are as follows: students' preferences (shocked as described above), colleges' preferences (estimated as described in Section 5.2), and colleges' quotas (set as described above).

6.3 Counterfactual Results

The counterfactual results are presented for three sets of outcomes of interest: (1) transfer outcomes, (2) students' preferences between mechanisms, and (3) colleges' imbalance. Transfer outcomes are either transfer with contract term I, transfer with term W, or stay at initial college (i.e., not transfer). Students' preferences are pairwise comparisons between mechanisms (Before, UCOP, and After) asking whether a student prefers their assignment from one mechanism to their assignment from another mechanism or is indifferent (receive the same assignment under both). Colleges' imbalance measures unfilled positions in a college's roster (i.e., exporting a student without filling that slot with an import). Imbalance is measured overall (college's number of imports is less than its exports) and separately in the short term (college's number of imports with term I is less than its exports with term I).

Figure 1 presents the transfer outcomes for the three mechanisms. Recall that the Before mechanism cannot assign students with a term-I contract to colleges other than her home college (which is why it is called COP-no-I) and that the After mechanism does not allow colleges to contest exports signing a term-I contract (i.e., colleges are unrestrictive, which is why this mechanism is called COP-Unr). The results clearly show that the Before mechanism results in fewer transfers (all of which are transfers with term W). The After mechanism results in a lot of transfers. UCOP is generally closer to the After mechanism.

In terms of students' preferences between mechanisms, Figure 2 compares Before to UCOP, Figure 3 compares After to UCOP, and Figure 4 compares Before to After. Very few students prefer Before to either other mechanism, which is consistent with the anecdotal evidence in Section 4 that suggests the pre-2021 transfer rules were inordinately restrictive in ways that hurt students' welfare. Except in 2015, between 60-80% of students strictly prefer UCOP to Before. Relative to After, between 30% and 60% of students strictly prefer After to UCOP with most of the remaining students receiving the same assignments from After and UCOP. It is

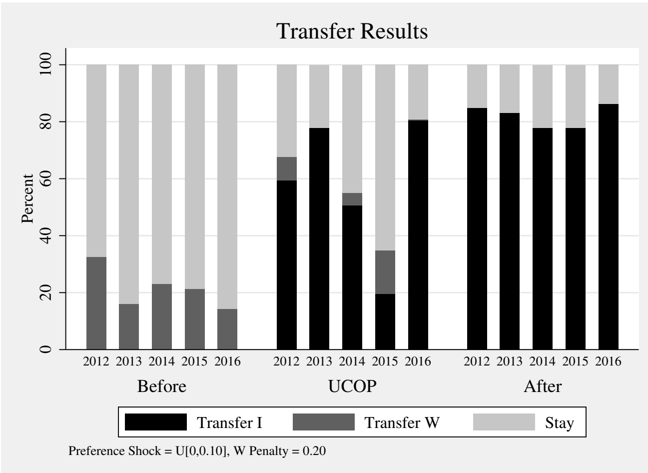

Figure 1: Transfer Results

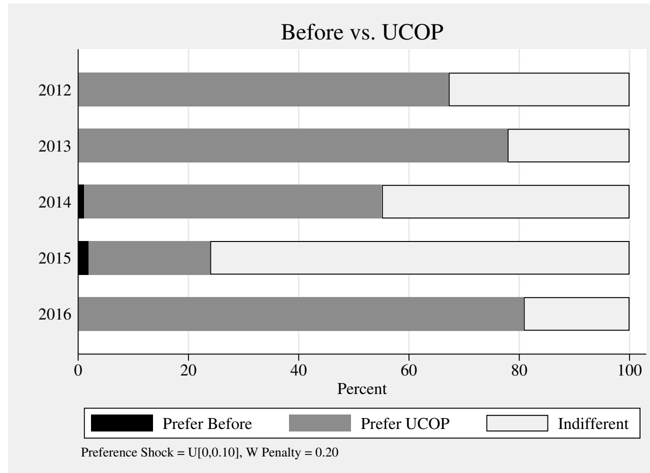

Figure 2: Students' Preferences: Before vs. UCOP

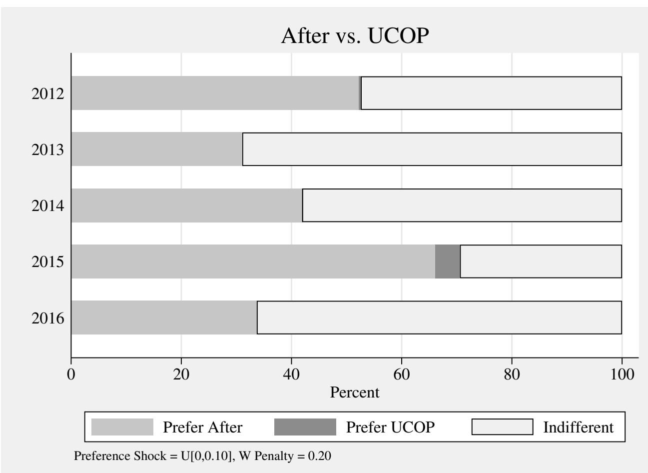

Figure 3: Students' Preferences: After vs. UCOP

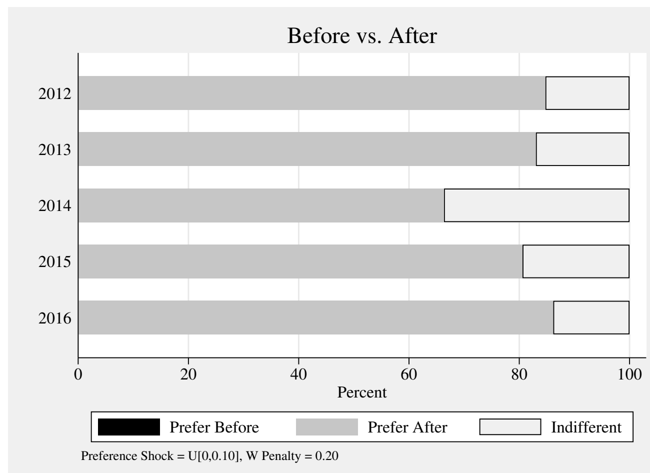

Figure 4: Students' Preferences: Before vs. After

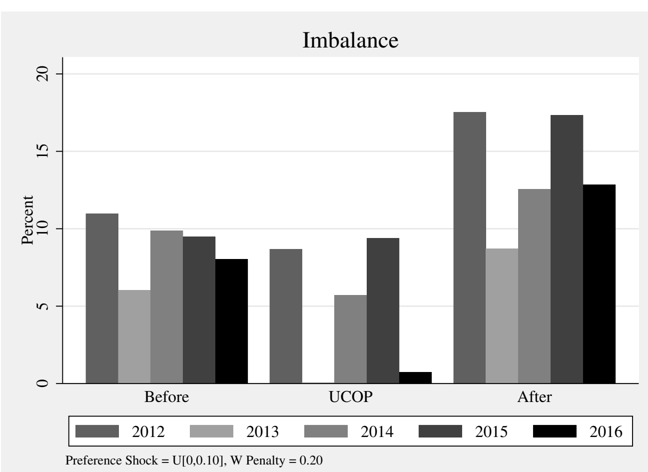

Figure 5: Colleges' Imbalance

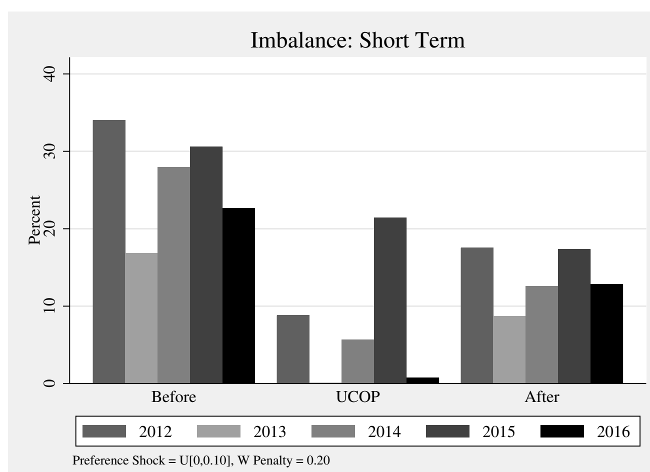

Figure 6: Colleges' Short-Term Imbalance

clear that if student welfare was the sole criterion, After is the preferred mechanism.

Turning to colleges, we use imbalance to measure how well colleges are doing. Colleges' imbalance shows how many unfilled positions are left after the transfer market runs compared to their initial roster size. Colleges with unfilled roster positions will have to expend additional costs and effort to recruit more players. Figure 5 shows imbalance overall; Figure 6 shows imbalance in the short term. Overall imbalance refers to colleges that did not receive a sufficient number of transfers, so these roster positions are unfilled. Short-term imbalance includes overall imbalance in its measure, and additionally, colleges that have transfers who cannot play for one year, so these roster positions are unfilled for the current year. The results are shown as the percentage of colleges with imbalance.

UCOP is much better than the other two mechanisms in terms of minimizing imbalance. UCOP features very little short-term imbalance: fewer than 10% of colleges have overall imbalance under UCOP and overall imbalance with UCOP is around 0% and 1% in 2013 and 2016, respectively. The Before mechanism is worst at minimizing short-term imbalance, while the After mechanism is worst at minimizing overall imbalance.

6.4 Counterfactual Robustness Checks

Two main elements determine the environment of interest in our analysis: the preference shock distribution and the W penalty. This subsection provides evidence that our results are robust to alternative choices. The preference shock distribution and W penalty are inherent features of students' preferences that are not clearly revealed by any of our data. As such, we set these parameters to most closely match the observed transfer rate in the data relative to the Before mechanism (which we used to emulate the decentralized market during the years in question). The alternative choices we present here are the other parameters that closely matched the data, though less closely fitting relative to those in the main results.

We demonstrate robustness to these parameters by showing results under nine alternative choices. Tables A1 through A54 present the six figures from the nine environments. They are as follows: preference shock distribution of $U[0, 0.01]$ and a W penalty of 0.20; preference shock distribution of $U[0, 0.01]$ and a W penalty of 0.25; preference shock distribution of $U[0, 0.05]$ and a W penalty of 0.20; preference shock distribution of $U[0, 0.05]$ and a W penalty of 0.25; preference shock distribution of $U[0, 0.10]$ and a W penalty of 0.25; preference shock distribution of $U[0, 0.15]$ and a W penalty of 0.05; preference shock distribution of $U[0, 0.15]$ and a W penalty of 0.10; preference shock distribution of $U[0, 0.15]$ and a W penalty of 0.15; preference shock distribution of $U[0, 0.15]$ and a W penalty of 0.20.[49] Comparing these nine sets of results to those in the main specification, we see remarkable similarities.

There are patterns in the results as the preference shock distribution becomes more dispersed; specifically, with a smaller preference shock upper bound, there are fewer transfers overall but a similar pattern of preferences across mechanisms. In all of these cases, the relative preference for UCOP stays quite similar. There are also patterns in the results as the W penalty increases; specifically, for a given shock distribution, there is less imbalance as the W penalty increases. Again, the relative performance of UCOP is similar in all nine alternative environments to that in the main environment from Section 6.3.

Overall, we see that the results of the transfer market differ only slightly depending on these modeling choices, which suggests that our comparison of UCOP to the Before and After mechanisms will be similar across a lot of alternative choices made during our analyses.

7 Discussions

7.1 Related Literature

The first class of papers related to ours inspects conditions similar to our balance condition in the outcome matching when there is an initial allocation, and rematching occurs based on this initial matching. Dur and Ünver (2019) introduced the balancedness condition in a two-sided matching market and proposed an efficient and student-strategy-proof mechanism in that setting, inspired by tuition exchange practices in the US colleges. Unlike our current paper, that paper focused on efficiency rather than stability. A similar effort was made in a market motivated for teacher assignment by Combe, Tercieux, and Terrier (2022) in France. Interestingly, this market currently uses a matching mechanism that is equivalent to the Before mechanism (COP-no-I mechanism) we use in our counterfactual analysis and theoretical analysis (Compte and Jehiel, 2008). Also, Guillen and Kesten (2012) observed that this mechanism is used in dormitory allocation and conducted experiments involving this mechanism and others (also see Abdulkadiroğlu and Sönmez, 1999). Another study in this vein is Combe, Dur, Tercieux, Terrier, and Ünver (2025) in the teacher reassignment market. It aims to design a mechanism for improving the equity in teacher quality distributions in different regions in France, while also improving the welfare of teachers, an objective different from ours. Hafalir, Kojima, and Yenmez (2022) and Kamada and Kojima (2025) inspect stable mechanisms under

[49]Recall that we selected ten parameter combinations (the main environment and nine alternative environments) based on their ability to fit the observed data. The best-fitting cases were characterized by higher disutility penalties (0.20 and 0.25), which appeared across all shock distribution ranges, and all four specifications with the widest shock distribution ($U[0, 0.15]$) fit well regardless of penalty level. The environment with the best fit features a preference shock distribution of $U[0, 0.10]$ with a W penalty of 0.20, and nearby specifications with similar parameter values also fit the data well. In contrast, the poor-fitting cases that were excluded tended to combine low-to-moderate shock distributions ($U[0, 0.01]$ or $U[0, 0.05]$) with low W penalties (less than 0.15). This suggests that the observed data are best explained by models that either feature large disutility from W contracts or highly dispersed preference shocks.

the balancedness condition as well, from different aspects.[50]

All the above studies differ from our work in one important way: these studies consider only fixed contractual terms. Therefore, the possibility of multiple contractual arrangements under balancedness conditions is inspected only in our paper. Without the balancedness condition, matching with the contracts model was introduced by Hatfield and Milgrom (2005).

Afacan (2024) focuses on the second of our two crucial features. He studies matching with status-quo assignments that require bilateral consent to dissolve and develops the notions of non-vetoing and conditional stability to characterize feasible reassignments under mutual-consent frictions without the balancedness requirement. The presence of initially unassigned agents is crucial for generating new matchings in his model.

Erdil and Ergin (2008, 2017) came up with algorithms to find an undominated-stable matching for school-choice and a Pareto-efficient stable matching for two-sided matching environments, respectively. The last stage of UCOP mechanism resembles these algorithms with the main exception that we have multiple contract possibilities in matches. Our Pareto improvements also focus on finding better uncontested contractual agreements given the same matches between agents. On the other hand, their algorithms are relevant when there are indifferences in the priorities of schools or preferences of agents. Moreover, in the Erdil and Ergin (2017) framework, it is feasible to find a Pareto efficient and stable matching repeatedly using *improving cycles* and *satisfying* such agents in the cycles in a two-sided matching market. However, in our environment, that is not always feasible.

Also, our paper belongs to a recent literature that uses field data to estimate agent preferences in matching markets. We use the methodology of Fack, Grenet, and He (2019) to estimate preferences. Two similar studies in this vein that use the same methodology are Combe, Tercieux, and Terrier (2022) and Combe, Dur, Tercieux, Terrier, and Ünver (2025).[51] Note that we need to estimate preferences from both sides of the market, while most papers in the related literature need to estimate only one side's preferences. A further distinguishing feature of our empirical analysis is that we study a setting in which agents on each side of the market do not submit a rank-ordered list of preferences over agents on the other side. As such, our empirical challenge is estimating preferences without preference lists using revealed preference data. A handful of other papers have also addressed the issue of how to estimate preferences in a matching market without data from an application system or full rank-ordered lists of preferences. Hitsch, Hortaçsu, and Ariely (2010) estimate preferences for men and women in an online dating market by using data on communication initiations within the platform. Related to their paper, Banerjee, Duflo, Ghatak, and Lafortune (2013) also estimate preferences for men

[50] There is a broader literature on distributional constraints—primarily regional—including but not limited to Kamada and Kojima (2015, 2017, 2018).

[51] See also Agarwal and Somaini (2018), who study school choice in elementary schools in Cambridge, MA, and Arslan (2021), who studies college admissions in Turkey, and Kamada, Kojima, and Matsushita (2025), who study the effect of integration of day-care districts on welfare.

and women, where they study the marriage market by using data on newspaper matrimonial advertisements.[52]

7.2 Other Applications and Concluding Remarks

We developed a novel matching model that can be used to address contractual designs regarding labor mobility in special markets, such as in team sports and foreign worker hiring. Although our data and main application are from NCAA college transfers, the same model and similar contractual framework can be used in markets for foreign worker employment in Malaysia, Saudi Arabia, Singapore, and Qatar, which employ a large foreign labor force with respect to their population.[53] They restrict foreign labor mobility, as domestic firms usually recruit such workers from abroad and take financial and legal responsibility for them. Secondary visa-worker transfer exists in an ad hoc manner in such countries if a firm does not object to the hiring of its foreign worker. Our framework brings a better-coordinated market design for these markets. Similarly, loan transfer markets in international association football regulated by Fédération Internationale de Football Association (FIFA) also fit as another application of our framework. In these markets, footballers are traded among professional clubs in secondary markets called loan markets without additional monetary terms. Primary trades among professional sports teams in North America may also fit our framework. In these markets, trades can be organized using mechanisms we propose even if there are no multiple contractual terms involved. Especially in our application, where a centralized college transfer portal already plays an important role in coordinating transfers, our remedy can be implemented quite easily.

Our incontestability axiom, together with a proper definition of stability, are used as the main solution desiderata in our mechanism design. When multiple contractual terms exist, our environment proves to be impossible to embed stability and strategy-proofness, and other desirable properties. Nevertheless, our proposed mechanism achieves no worse strategic performance with respect to any other mechanism that could Pareto dominate it.

Our study also distinguishes itself from similar studies by its novel empirical estimation and counterfactual analysis using collected historical NCAA data. While there are recent studies that estimate agent preferences using submitted rank-order lists, our study is one of the few that uses revealed preference data and uses a different empirical estimation technique from this latter class of papers, due to Fack, Grenet, and He (2019). In our counterfactual analysis, we observed significant gains in our proposed mechanism for colleges with respect to either

[52]The econometric models in our analyses differ from these papers. Specifically, Hitsch, Hortaçsu, and Ariely (2010) estimate a fixed effects logit model, while Banerjee, Duflo, Ghatak, and Lafortune (2013) estimate a conditional logit model with fixed effects as well as an OLS model with fixed effects. In contrast, we adapt the Fack, Grenet, and He (2019) econometric model to fit our setting.

[53]See https://www.fragomen.com/insights/the-immigration-impact-of-changing-employers-in-malaysia.html, https://www.aljazeera.com/news/2021/3/14/saudi-arabias-long-awaited-kafala-reform-goes-into-effect, https://www.mom.gov.sg/passes-and-permits/work-permit-for-foreign-worker/sector-specific-rules/hiring-existing-worker-in-process-sector, https://qatarofw.com/changing-employers.

NCAA contractual environment. At the same time, students' welfare goes up with respect to the pre-2021 environment and not as high as the post-2020 environment. However, we should caution that ours is a conservative estimate as we did not take into account the welfare of new college recruits from high schools. Anecdotal evidence suggests that scholarship offers to these students became more restrictive after 2021. This can be a further avenue of empirical research.

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A Omitted Results and Proofs

Lemma 1 *$\mathcal{C}_c$ satisfies IRC, substitutability and LAD.*

Proof. IRC: For any $X' \subseteq X$, in each step of the calculation of $\mathcal{C}_c$ we consider the most preferred contract among the remaining ones. Then, the removal of an unchosen contract does not affect the set of chosen ones.

Substitutability: On the contrary, suppose there exists a set of contracts X' such that $x \notin \mathcal{C}_c(X' \cup \{x\})$ but $x \in \mathcal{C}_c(X' \cup \{x, y\})$ for some $x, y \in X$.

For x to be chosen in some step k of the calculation of $\mathcal{C}_c(X' \cup \{x, y\})$, y needs to be accepted in some step $k' < k$. Hence, under $\mathcal{C}_c(X' \cup \{x, y\})$ and $\mathcal{C}_c(X' \cup \{x\})$, the same contracts are chosen and removed in the first $k' - 1$ steps. Moreover, the student in the contract chosen in step $k' + 1$ of the calculation of $\mathcal{C}_c(X' \cup \{x, y\})$ is a contract of either the student assigned in step k' or $k' + 1$ of the calculation of $\mathcal{C}_c(X' \cup \{x\})$. Hence, $x \notin \mathcal{C}_c(X' \cup \{x, y\})$.

Law of Aggregate Demand: On the contrary, suppose there exists a set of contracts X' and a contract $x \notin X'$ such that $\left|\mathcal{C}_c(X' \cup \{x\})\right| < \left|\mathcal{C}_c(X')\right|$. If $x \notin \mathcal{C}_c(X' \cup \{x\})$, then $\mathcal{C}_c(X' \cup \{x\}) = \mathcal{C}_c(X')$ by definition. Suppose $x \in \mathcal{C}_c(X' \cup \{x\})$ and it is chosen in Step k'

under the calculation of $\mathcal{C}_c(X' \cup \{x\})$. As explained above, the student in the contract chosen in step $k'+1$ of the calculation of $\mathcal{C}_c(X' \cup \{x\})$ is a contract of either the student assigned in step k' or $k'+1$ of the calculation of $\mathcal{C}_c(X')$. Hence, $\left|\mathcal{C}_c(X' \cup \{x\})\right| \geq \left|\mathcal{C}_c(X')\right|$. ■

Proof of Proposition 1. We first show the result for the case in which colleges are unrestrictive.

Part 1: $C^R = \emptyset$**.**

We first prove the stability of COP-Unr. Recall that when colleges are unrestrictive, any matching is uncontested, and stability only requires individual rationality and elimination of blocking pairs. Let μ be the outcome of COP-Unr. Assumption 1 (i) and (ii) and our construction of auxiliary college choice rules imply that each student s is matched with a contract weakly higher ranked than his initial contract $(s, \omega(s), I)$ under P_s. Moreover, by definition of COP-Unr each agent is matched with an acceptable contract. Hence, μ is individually rational. If $(s, c, t)\ P_s\ \mu_s$ for some s, c, and t, then by the definition of COP-Unr, college c has rejected (s, c, t) during the execution of COP stage of COP-Unr using the auxiliary choice rule $\widehat{\mathcal{C}}_c$. Thus, whenever there exists a student-college pair (s, c) such that $(s, c, t)\ P_s\ \mu_s$, either $(s, c, t) \notin \mathcal{C}_c(\mu_c \cup \{(s, c, t)\})$ or there exists $(s', c, I) \in \mu_c \setminus \mathcal{C}_c(\mu_c \cup \{(s, c, t)\})$ for some $s' \in S_c$.

Next, we prove the student-strategy-proofness of COP-Unr. Lemma 1 shows that $\mathcal{C}_c$ is substitutable and satisfies LAD. Since this result does not depend on the preference relation inducing $\mathcal{C}_c$, $\widehat{\mathcal{C}}_c$ is substitutable and satisfies LAD. Since COP is strategy-proof for choice rules satisfying substitutability and LAD (see Hatfield and Milgrom, 2005), COP-Unr is immune to strategic manipulations by students.

Part 2: $C^R = C$**.**

We first prove the stability of COP-no-I. Let μ be the outcome of COP-no-I. Assumption 1 (i) and (ii) and our construction of $\widehat{P}$ imply that each student s is matched with a contract weakly higher ranked than his initial contract $(s, \omega(s), I)$ under both P_s and $\widehat{P}_s$. By our construction of students' modified preferences, $\mu_c^e \cap \mu_I = \mu_c^m \cap \mu_I = \emptyset$ for each college c. Moreover, by the definition of COP-no-I, each agent is matched with acceptable contracts. Hence, μ is uncontested and individually rational. If $(s, c, W)\ P_s\ \mu_s$ for some s and c, then by the definition of COP and Assumption 1 (i)[54] college c has rejected (s, c, W) during the execution of COP for market $\widehat{P}$. Then, $(s', c, t)\ P_c\ (s, c, W)$ and $(s', c, t)\ \widehat{P}_c\ (s, c, W)$ for all $(s', c, t) \in \mu_c$ and $s' \notin S_c$. Thus, if there exists student-college pair (s, c) and term t such that $(s, c, t)\ P_s\ \mu_s$, $(s, c, t) \in \mathcal{C}_c\big(\mu_c \cup \{(s, c, t)\}\big)$ and there does not exist $(s', c, I) \in \mu_c \setminus \mathcal{C}_c\big(\mu_c \cup \{(s, c, t)\}\big)$ for some $s' \in S_c$, then our construction of $\widehat{P}_s$, $\widehat{P}_c$ and $\widehat{\mathcal{C}}_c$ imply that $t = I$. Then under $\nu = \big(\mu \setminus (\mu_c \cup \mu_s)\big) \cup \mathcal{C}_c\big(\mu_c \cup \{(s, c, t)\}\big)$, we have $\nu^e_{\omega(s)} \cap \nu_I = \{(s, c, I)\}$ but $\nu^m_{\omega(s)} \cap \nu_I = \emptyset$. Hence, (s, c) cannot be a blocking pair. Moreover, given that all students can apply with one

[54] Assumption 1 (i) eliminates the case in which s is matched with $(s, \omega(s), I)$ but prefers $(s, \omega(s), W)$ over $(s, \omega(s), I)$.

contract to each college, the choice rules with all applicable offers satisfy substitutability and LAD.[55] Since COP is strategy-proof for choice rules satisfying substitutability and LAD (see Hatfield and Milgrom, 2005) and for each student s, $\widehat{P}_s$ respects true order of contracts with term W and $(s, \omega(s), I)$ under P_s, COP-no-I is immune to strategic manipulations by students. ■

Proof of Proposition 2. We first prove that COP-Unr is constrained efficient. Consider an arbitrary market P. Let μ be the outcome of COP-Unr. By Proposition 1, μ is stable. On the contrary, suppose there exists another matching ν, which is partner equivalent to μ and Pareto dominates μ. Then, there exists at least one college-student pair (c, s) such that $\mathrm{c}(\mu_s) = \mathrm{c}(\nu_s) = c, \quad \nu_s \ P_s \ \mu_s, \quad$ and $\nu_c \ P_c \ \mu_c$. By Assumption 1 (i), $\mathrm{t}(\nu_s) = I$ and $\mathrm{t}(\mu_s) = W$. Then, stability of μ implies that $c \neq \omega(s)$. Moreover, college c has rejected ν_s when COP-Unr is applied to market P. Then, responsiveness of the college choice rule implies that $\mu_s \ P_c \ \nu_s$. Since these observations hold for any student preferring ν to μ, $\mu_c \ P_c \ \nu_c$. This is a contradiction.

In Example 3, we show that when colleges are unrestrictive or restrictive, there does not exist a strategy-proof and undominated stable mechanism. Since COP-Unr is strategy-proof, it cannot be undominated stable (and therefore Pareto efficient). ■

Proof of Theorem 1. We prove the theorem through an example. There are 3 colleges $C = \{a, b, c\}$ with capacities $q_a = 2$, $q_b = 1$, and $q_c = 1$. Colleges are restrictive. Let $S_a = \{s, s'\}$, $S_b = \{s''\}$ and $S_c = \emptyset$. Preferences satisfy Assumption 1 and are given as:

P_s	$P_{s'}$	$P_{s''}$	P_a	P_b	P_c
(s, c, I)	(s', b, I)	(s'', c, I)	(s'', a, I)	(s, b, I)	(s, c, I)
(s, b, I)	(s', b, W)	(s'', c, W)	(s, a, I)	(s', b, I)	(s', c, I)
(s, c, W)	(s', a, I)	(s'', a, I)	(s', a, I)	(s'', b, I)	(s'', c, I)
(s, b, W)	$\vdots$	(s'', a, W)	$\vdots$	(s, b, W)	(s, c, W)
(s, a, I)		(s'', b, I)		(s', b, W)	(s', c, W)
$\vdots$		$\vdots$		(s'', b, W)	(s'', c, W)
				$x_\emptyset$	$x_\emptyset$

First, we show the following:

Claim: In any stable matching, s or s'' is matched to college c with term W.

Proof: Suppose to the contrary that neither s nor s'' is matched to c with the term W in any stable matching. Then by individual rationality of a stable matching, as s' finds c unacceptable under any contract, no student is matched with c with term W. If a student is matched with c with term I, then such a matching cannot be uncontested as either a or b loses one net student,

[55] As explained in Case 1, $\widehat{\mathcal{C}}_c$ is substitutable and satisfies LAD.

who is matched with term I. Then, no student is matched with c in any stable matching. As a result, (s'', c) blocks this matching with term W. This is a contradiction. QED

Also, in any stable matching, s' is matched to a or b by individual rationality.

Suppose μ is stable and $\mu_{s''} = (s'', c, W)$. For (s, c) not to block μ with term W, s needs to be matched to contract (s, b, I). Then by individual rationality and immediate eligibility of a student at his initial college, s' needs to be matched to contract (s', a, I). Since $|\mu_a^e \cap \mu_I| = 1$ and $|\mu_a^m \cap \mu_I| = 0$, μ is contested by a, contradicting stability. Hence, a matching in which s'' is matched to contract (s'', c, W) cannot be stable.

Suppose μ is stable and $\mu_s = (s, c, W)$. Three cases are possible for s'':

If s'' is matched to his home college b: Pair (s'', a) blocks this matching with term W, as a has only s' matched in μ while $q_a = 2$. A contradiction occurs.

If s'' is matched to a with term I: Then s' should be matched to b with term I so b does not contest μ. However, pair (s, b) blocks this matching with term I. A contradiction occurs.

If s'' is matched to a with term W. Then s' should be matched to b with term W so a does not contest μ. However, μ is Pareto dominated by uncontested matching ν such that

$$\nu = \{(s, c, W),\ (s', b, I),\ (s'', a, I)\}$$

and μ and ν are partner-equivalent. This concludes the proof. ■

Proof of Theorem 2. Consider an arbitrary market P. We first show that in every repetition of Stage 3, we update a student's preferences by moving a contract with term I below $x_\emptyset$. First of all, whenever the matching obtained in Stage 2 is contested (Case 1), by the definition of the mechanism, we move a contract with term I below $x_\emptyset$. Hence, we focus on the applications of Case 2 of Stage 3. Consider the first application of Case 2 of Stage 3. Every acceptable contract with term W under P_s for some student s is also acceptable under $\overline{P}_s$. Suppose a student-college pair (s, c) blocks μ with term t. Then, we claim that $|\mu_c \setminus \mathcal{C}_c(\mu_c \cup \{(s, c, t)\})| = 1$. On the contrary, suppose $|\mu_c \setminus \mathcal{C}_c(\mu_c \cup \{(s, c, t)\})| \neq 1$. By the definition of choice function $\mathcal{C}_c$, we cannot have $|\mu_c \setminus \mathcal{C}_c(\mu_c \cup \{(s, c, t)\})| > 1$. Then, suppose $|\mu_c \setminus \mathcal{C}_c(\mu_c \cup \{(s, c, t)\})| = 0$. We consider two subcases: $t = W$ and $t = I$. If $t = W$, then (s, c, W) was rejected by c in Stage 2. The construction of the auxiliary choice function implies that $|\mu_c| = q_c$ and $x \ P_c \ (s, c, W)$ for all $x \in \mu_c$ such that $\mathrm{s}(x) \in S_c$. Then, the definition of blocking pair implies that (s, c) cannot block μ with term W. If $t = I$, then $s \notin S_c$ and adding (s, c, I) to μ would violate incontestability.

Let $\tilde{x} = \mu_c \setminus \mathcal{C}_c\big(\mu_c \cup \{(s, c, t)\}\big)$. Recall that, (s, c) cannot block μ with term W and $s \notin S_c$. Hence, by the definition of blocking pair, $\mathrm{t}(\tilde{x}) = I$. Since in every application of Case 2 of Stage 3, all these arguments hold, whenever we update a student's preferences in Stage 3, we move a contract with term I below $x_\emptyset$. In other words, every acceptable contract with term W under P_s for any student s is also acceptable under $\overline{P}_s$.

Next, we show that UCOP terminates under market P. Since the sets of colleges and students are finite, we can repeat Stages 2 and 3 of UCOP finitely many times. In the extreme case, for every $s \in S$, $(s, \omega(s), I)$ would be the only acceptable term-I contract under the updated preference relation obtained in the last application of Stage 3 and all acceptable contracts with term W under the original preferences will be acceptable under the updated preferences for all students. Then, the proof of Proposition 1 implies that the outcome obtained in the last application of Stages 2 and 3 does not include any blocking pair and is uncontested. Moreover, the finiteness of the sets of students and colleges implies that there are finitely many improvement cycles in Stage 4. Hence, the mechanism terminates. Let $\overline{\mu}$ be the outcome of UCOP under P. We denote the outcome obtained in the last application of Stages 2 and 3 by μ. That is, $\mu \neq \overline{\mu}$ if we implement improvement cycles in Stage 4.

Next, we show that $\overline{\mu}$ is stable.

$\overline{\mu}$ is uncontested: By the definition of Stage 3, μ is uncontested, i.e., $|\mu_c^e \cap \mu_I| = |\mu_c^m \cap \mu_I|$ for every $c \in C$. Finally, in each improvement cycle execution in Stage 4, the numbers of exported and imported term-I contracts are equal for every college. Hence, $\overline{\mu}$ is uncontested.

$\overline{\mu}$ individually rational: In Stage 1 for each $s \in S$, we move contract $(s, \omega(s), I)$ above all contracts of students in $S \setminus S_{\omega(s)}$ with college $\omega(s)$ and construct $\overline{P}_{\omega(s)}$. By the definition of $\overline{C}_{\omega(s)}$, $(s, \omega(s), I) \in \overline{C}_{\omega(s)}(X')$ for any $(s, \omega(s), I) \in X'$. Hence, whenever a student s applies to $\omega(s)$ with contract $(s, \omega(s), I)$ in some repetition of Stage 2, $(s, \omega(s), I)$ will be accepted by $\omega(s)$ and will not be rejected in the further steps of Stage 2. Moreover, for any $s \in S$, we never remove contract $(s, \omega(s), I)$ from student s's preferences in some repetition of Stage 3 due to a violation of incontestability or the existence of a blocking pair. Hence, $\mu_s \; R_s \; (s, \omega(s), I)$ for all $s \in S$. By Assumption 1 (i), Stage 4 improves the welfare of students (and colleges as long as they prefer contracts with term I over contracts with term W). Hence, for any market, UCOP matches each student $s \in S$ with a contract weakly better than $(s, \omega(s), I)$. By Assumption 1 (ii) under $\overline{\mu}$, every student is matched with an acceptable contract.

When a student s applies to a college c with an unacceptable contract under P_c, he will be rejected in Stage 2. In neither Stage 2 nor Stage 4 (by Assumption 1 (iv)) will a college be matched with an unacceptable contract. Hence, $\overline{\mu}$ is individually rational.

No blocking pair in $\overline{\mu}$: Since we repeat Stage 2 and Stage 3 until we get a matching without blocking pairs, there is no blocking pair in μ. The conditions that determine whether or not a student can point to her assigned college prevent the existence of a blocking pair after implementing cycles. By Assumption 1 (iii), since each cycle selection in Stage 4 preserves stability, μ is stable.

This completes the proof. ■

Proof of Proposition 3. Consider an arbitrary market P such that $q_c \leq 1$ for all $c \in C = C^R$. Let $\overline{\mu}$ be the outcome of UCOP under market P. Theorem 2 implies that $\overline{\mu}$ is stable. On the contrary, suppose $\overline{\mu}$ is not constrained efficient, i.e., there exists another uncontested matching ν such that $\overline{\mu}$ and ν are partner-equivalent and ν Pareto dominates $\overline{\mu}$. Hence, $\mathrm{t}(\nu_s) = I$ and $\mathrm{t}(\mu_s) = W$ for any s with $\mu_s \neq \nu_s$. Moreover, $\mathrm{s}(\mu_c) \in S$ if and only if $\mathrm{s}(\nu_c) \in S$ for every $c \in C$. Since μ and ν are uncontested, if $\mu_s \neq \nu_s$, then $\mathrm{t}(\mu_{\omega(s)}) = W$ and $\mathrm{t}(\nu_{\omega(s)}) = I$. Hence, among the students who are better off under ν compared to μ, there exists at least one improvement cycle where each student points to his assigned college under μ and ν, and each college points to its initial student. When $q_c \leq 1$ for all c, in Stage 4 of UCOP every student s matched to contract x at the end of Stage 3 where $\mathrm{t}(x) = W$ points to $\mathrm{c}(x)$. Hence, such a cycle should have been constructed under Stage 4 of UCOP. This is a contradiction. ■

Proof of Theorem 3. We prove this result via an example. There are four colleges $C = \{a, b, c, d\}$, each with a capacity of one. Colleges are restrictive. Let $S_a = \{s\}$, $S_b = \{s'\}$, $S_c = \{s''\}$, and $S_d = \emptyset$. Preferences are given as:

P_s	$P_{s'}$	$P_{s''}$	P_a	P_b	P_c	P_d
(s,c,I)	(s',c,I)	(s'',d,I)	(s,a,I)	(s',b,I)	(s'',c,I)	(s',d,I)
(s,c,W)	(s',b,I)	(s'',d,W)	(s,a,W)	(s',b,W)	(s'',c,W)	(s',d,W)
(s,d,I)	$\vdots$	(s'',b,I)	$\vdots$	(s'',b,I)	(s',c,I)	(s,d,I)
(s,d,W)		(s'',b,W)		(s'',b,W)	(s',c,W)	(s,d,W)
(s,a,I)		(s'',c,I)		$\vdots$	(s,c,I)	(s'',d,I)
$\vdots$		$\vdots$			(s,c,W)	(s'',d,W)
					$\vdots$	$\vdots$

We refer to this market as the original market. First, notice that, since s' and s'' rank contracts with college a below their initial contracts, in a stable matching, no student other than s can be matched with a. As a result, in a stable matching, s cannot be matched with college other than $\omega(s) = a$ with term I. Observation 1 implies that no student can be matched with college d with term I. Moreover, either student s or s'' is matched with college d with term W under any stable matching. As a result, there are three stable matchings in this market:

$$\mu^1 = \{(s,c,W), (s',b,I), (s'',d,W)\}$$
$$\mu^2 = \{(s,d,W), (s',b,I), (s'',c,I)\}$$
$$\mu^3 = \{(s,d,W), (s',c,I), (s'',b,I)\}$$

There is no Pareto ranking between these three matchings. Moreover, there does not exist another uncontested partner-equivalent matching that Pareto dominates any of these stable matchings. That is, all these matchings are constrained efficient.

Consider any stable mechanism selecting μ^2 or μ^3. Suppose s submits

$$P'_s : \quad (s,c,I)\ P'_s\ (s,c,W)\ P'_s\ (s,a,I) \dots$$

Similar to the situation in market P, s cannot be assigned to c with term I under a stable matching in market (P'_s, P_{-s}). Then s will be either matched with (s,c,W) or (s,a,I). If the former case holds, this is a profitable manipulation for s. Suppose s is assigned to (s,a,I). Then for s'' and d to not block with term W, s'' will be assigned to (s'',d,W) and s' will be assigned to (s',b,I). However, this matching is blocked by s and c with term W. That is, a stable and strategy-proof mechanism cannot select μ^2 or μ^3 in the original market.

Now consider any stable and constrained-efficient mechanism that selects μ^1 under the original market. Suppose s' submits

$$P'_{s'} : \quad (s',c,I)\ P'_{s'}\ (s',c,W)\ P'_{s'}\ (s',d,I)\ P'_{s'}\ (s',d,W)\ P'_{s'}\ (s',b,I) \dots$$

Under any stable matching, s' is assigned to a contract no worse than (s',d,W), as otherwise s' and d would block such a matching with term W. If he is assigned to (s',c,I), then this is a profitable manipulation for s'. The matching is contested if he is assigned to (s',d,I). Two possible cases remain.

1. Suppose s' is assigned to (s',d,W). Then for s' and c to not form a blocking pair with term W, s'' needs to be assigned to (s'',c,I) and s needs to be assigned to (s,a,I). However, s'' and b blocks this matching with (s'',b,W).
2. Suppose s' is assigned to (s',c,W). Then s will be assigned to (s,d,W) and s'' will be assigned to (s',b,W). This matching is stable but is Pareto dominated by another stable matching, namely μ^2 that is partner-equivalent to this matching. Hence, this matching cannot be constrained efficient.

This concludes the proof. ■

Proof of Proposition 4. We prove this result via an example. There are four colleges $C = \{a,b,c,d\}$, each with a capacity of one. Colleges are restrictive. Let $S_a = \{s\}$, $S_b = \{s'\}$, $S_c = \emptyset$, and $S_d = \{s''\}$. Preferences are given as:

P_s	$P_{s'}$	$P_{s''}$	P_a	P_b	P_c	P_d
(s,b,I)	(s',c,I)	(s'',a,I)	(s',a,I)	(s'',b,I)	(s'',c,I)	$\vdots$
(s,b,W)	(s',c,W)	(s'',a,W)	(s',a,W)	(s'',b,W)	(s'',c,W)	
(s,a,I)	(s',a,I)	(s'',c,I)	(s'',a,I)	(s,b,I)	(s,c,I)	
$\vdots$	(s',a,I)	(s'',c,W)	(s'',a,W)	(s,b,W)	(s,c,W)	
	(s',b,I)	(s'',d,I)	$\vdots$	(s',b,I)	(s',c,I)	
	$\vdots$	$\vdots$		(s',b,W)	(s',c,W)	
				$x_\emptyset$	$x_\emptyset$	

We refer to this market as the original market. Under the original market, UCOP selects matching

$$\mu = \{(s,b,W),(s',c,W),(s'',a,W)\}.$$

There is no uncontested matching that Pareto dominates μ.

Let ϕ be an uncontested, strategy-proof mechanism that Pareto dominates UCOP. Then, ϕ selects μ in this market.

Suppose s submits

$$P'_s: \quad (s,c,I)\ P'_s\ (s,c,W)\ P'_s\ (s,b,I)\ P'_s\ (s,b,W)\ P'_s\ (s,a,I)\ldots$$

When all the other agents submit their true preferences, UCOP selects matching

$$\mu' = \{(s,b,I),(s',a,I),(s'',c,W)\}.$$

There is no uncontested matching that Pareto dominates μ'. Hence, ϕ selects μ' and student s gains from misreporting under the original market. As a result, ϕ is not strategy-proof. ■

Proof of Proposition 5. We prove this result via an example in which students cannot manipulate UCOP, but any mechanism that Pareto improves UCOP can be manipulated. There are three colleges $C = \{a,b,c\}$, each with a capacity of one. Let $S_a = \{s\}$, $S_b = \{s'\}$, and $S_c = \{s''\}$. Preferences are given as:

P_s	$P_{s'}$	$P_{s''}$	P_a	P_b	P_c
(s,b,I)	(s',a,I)	(s'',b,I)	(s',a,I)	(s'',b,I)	(s'',c,I)
(s,c,I)	(s',b,I)	(s'',c,I)	(s',a,W)	(s'',b,W)	(s'',c,W)
(s,a,I)	(s',c,I)	(s'',a,I)	(s,a,I)	(s,b,I)	(s',c,I)
$\vdots$	$\vdots$	$\vdots$	(s,a,W)	(s,b,W)	(s',c,W)
			(s'',a,I)	(s',b,I)	(s,c,I)
			(s'',a,W)	(s',b,W)	(s,c,W)
			$x_\emptyset$	$x_\emptyset$	$x_\emptyset$

We refer to this market as the original market. Under the original market, UCOP selects matching

$$\mu = \{(s,c,I),(s',a,I),(s'',b,I)\}.$$

There is no matching Pareto dominating μ. Hence, any mechanism that Pareto improves UCOP selects μ under this market. Moreover, no student can gain from misreporting his preferences under UCOP in this market. To see this point, first notice that s' and s'' are assigned to their first choices. As a result, s' and s'' do not have an incentive to misreport under UCOP in this market. We consider student s. If s reports either (s,a,I) or (s,c,I) as his top choice, then UCOP will assign him to his reported top choice. If s reports (s,b,I) and (s,a,I) as the top two choices, then UCOP will assign him to (s,a,I). If s reports (s,b,I) and (s,b,W) as the top two choices, then UCOP will assign him to his third choice, which is either (s,a,I) or (s,c,I).[56]

Suppose s submits

$$P_s': \quad (s,b,I)\ P_s'\ (s,a,I)\ P_s'\ (s,c,I)\dots$$

When all the other agents submit their true preferences, UCOP selects matching

$$\mu' = \{(s,a,I),(s',b,I),(s'',c,I)\}.$$

Matching μ' is Pareto inefficient. The unique matching Pareto dominating μ' in market (P_s', P_{-s}) is

$$\nu = \{(s,b,I),(s',a,I),(s'',c,I)\}.$$

Hence, any mechanism that Pareto improves UCOP selects ν under this market. As a result, student s gains from misreporting his preferences under any mechanism Pareto improving UCOP whenever possible under the original market. ■

Proof of Theorem 4. Consider a market P such that $q_c = |S_c|$ for all $c \in C$. Let μ be the outcome of UCOP under market P and $\overline{\mathcal{C}}_c$ be the choice function induced by preference order

[56]Recall that, under Assumption 1, every student $\bar{s}$ prefers $(\bar{s},\bar{c},I)$ to $(\bar{s},\bar{c},W)$ for all $\bar{c} \in C$ and $(\bar{s},\omega(\bar{s}),I)$ to $x_\emptyset$.

$\overline{P}_c$ for all $c \in C$.

We first show that the matching selected at the end of the first application of the COP mechanism is stable, and therefore, the UCOP mechanism terminates at the end of the first application of Stage 2.

Let μ be the outcome of the first application of COP mechanism. Assumption 1, the constructions of $\overline{P}_c$ and $\overline{\mathcal{C}}_c$, and $q_c = |S_c|$ for for all $c \in C$ imply that $\mu_s \ R_s \ \omega_s$, $x \ P_c \ x_\emptyset$, and $|\mu_c| = q_c$ for all $x \in \mu_c$, $s \in s$, and $c \in C$. Hence, μ is individually rational.

Next, we claim that $\mathrm{t}(\mu_s) = I$ for all $s \in S$. In particular, during the application of COP, no college tentatively accepts a contract with term W. On the contrary, suppose some college c tentatively accepts a contract x with $\mathrm{t}(x) = W$ in Step k of COP. Without loss of generality, let k be the earliest step of COP in which a contract with term W, say x, is tentatively accepted by some college. Let $x = (s, c, W)$. By Assumption 1, in Step 1 of COP, all students propose with term-I contracts. Hence, $k > 1$. Moreover, Assumption 1 implies that s has proposed to c with contract (s, c, I) in some Step $k' < k$ and is rejected in some Step k'' such that $k > k'' \geq k'$. By the definition of $\overline{\mathcal{C}}_c$ and Assumption 1, in Step k'', c is holding q_c term-I contracts and all these contracts are ranked above (s, c, I) under $\overline{P}_c$. In all following steps, each tentatively accepted contract by c is ranked above (s, c, I) under $\overline{P}_c$. Assumption 2 and the construction of $\overline{P}_c$ from P_c imply that (s, c, W) is ranked below all contracts tentatively held in Step $k - 1$. Hence, c cannot tentatively accept (s, c, W) in Step k.

Since $|\mu_c| = q_c$ for all $c \in C$ and $\mathrm{t}(\mu_s) = I$ for all $s \in S$, μ is uncontested.

By the construction of $\overline{P}_c$ and $\overline{\mathcal{C}}_c$, if there exists a student s such that $(s, c, t) \ P_s \ \mu(s)$, then $\mu(s') \ P_c \ (s, c, t)$ for all $\mu_{s'} \in \mu_c$ where $s' \notin S_c$. As a result, if there exists a college-student pair (c, s) and term t such that $(s, c, t) \ P_s \ \mu_s$ and $(s, c, t) \in \mathcal{C}_c\big(\mu_c \cup \{(s, c, t)\}\big)$, then there exists $(s', c, I) \in \mu_c$ and $(s', c, I) \notin \mathcal{C}_c\big(\mu_c \cup \{(s, c, t)\}\big)$ such that $\omega(s') = c$. Hence, such a college-student pair cannot be a blocking pair, and μ is stable.

Since all contracts accepted by colleges are with term I, Assumption 1 implies that μ is constrained efficient.

Lemma 1 implies that $\overline{\mathcal{C}}_c$ is substitutable and satisfies LAD for all $c \in C$. Since we consider P_S in the calculation of COP in Stage 2, by Hatfield and Milgrom (2005), no student can manipulate UCOP, i.e., UCOP is strategy-proof for students. ■

Proof of Proposition 6. Consider an arbitrary market such that $q_c = |S_c| = 1$ for all $c \in C$. Suppose Assumption 1 holds. Let μ be the outcome of TTC in this market. Since each student s is pointed to by his initial college $\omega(s)$, Assumption 1 implies that s is matched with a contract weakly better than $\big(s, \omega(s), I\big)$. Moreover, by definition, each college is matched with an acceptable contract. Assumption 1 implies that if a student can point to college c, then (s, c, I) is acceptable to c. As a result, each student in a selected cycle in some Step k is matched with a term-I contract. Hence, μ is individually rational and uncontested. Since $|S_c| = 1$ for all

$c \in C$ and all students are matched with term-I contract under μ, we may have a blocking pair (s, c) only if s is matched with $\omega(\mathrm{s}(\mu_c))$. However, $\mathrm{s}(\mu_c)$ and s are removed in the same cycle and $(s, \omega(\mathrm{s}(\mu_c)), I)\ P_s\ (s, c, I)$. Hence, μ is stable.

Let χ_k be the set of students in the cycles selected in Step k of TTC-wC. By induction, we show that TTC-wC is Pareto efficient and strategy-proof for students. We first prove Pareto efficiency. We start with χ_1. Every student in χ_1 is assigned to his most preferred contract among the ones that are acceptable to the corresponding colleges. Hence, we cannot assign students in χ_1 to a more preferred contract without assigning an unacceptable contract to a college and hurting that college.

Suppose that for every $k' < k$, every student $s \in \chi_{k'}$ cannot be assigned to a better contract without hurting some college c or a student in $\chi_{\tilde{k}}$ where $\tilde{k} < k'$. We consider students in χ_k. By definition, every student $s \in \chi_k$ is assigned to the best contract in Step k among the contracts with remaining colleges and acceptable to the corresponding colleges. Hence, if we assign s to a better contract x such that $\mathrm{c}(x)$ is a remaining college in Step k, then x is unacceptable for $\mathrm{c}(x)$. If we assign s to a better contract x such that $\mathrm{c}(x)$ is removed in some Step $\tilde{k} < k$, then this will hurt some other student in $\chi_{\tilde{k}}$. Therefore, any student $s \in \chi_k$ cannot be assigned to a better contract without hurting some college c or a student in $\chi_{\tilde{k}}$ where $\tilde{k} < k$. This concludes that μ is Pareto efficient.

Next, we prove strategy-proofness of TTC-wC. We consider students in χ_1. Recall that, every student in χ_1 is assigned to his most preferred contract among the ones that are acceptable to the corresponding colleges. Since TTC-wC is individually rational, no student in χ_1 can be assigned to a better contract, i.e., a contract that is unacceptable for the corresponding college, by misreporting his preferences.

Suppose that for every $k' < k$, any student $s \in \chi_{k'}$ cannot be assigned to a better contract by misreporting. We consider the students in χ_k. By following the first $k-1$ steps of TTC-wC one by one, we can easily show that a student in χ_k cannot affect the cycles formed in an earlier step. As a result, any student $s \in \chi_k$ cannot be assigned to contract x such that college $\mathrm{c}(x)$ is removed in an earlier step by misreporting. Since TTC-wC is individually rational, no student in χ_k can be assigned to a contract x such that $\mathrm{c}(x)$ considers x unacceptable. Hence, no student in χ_k can be assigned to a better contract by misreporting his preferences. ■

B Omitted Examples

We show, via Example 4, that the existence of a stable matching cannot be guaranteed when we have both restrictive and unrestrictive colleges in a market.

Example 4 *There are four colleges, $C = \{a, b, c, d\}$, and three students, $S = \{s, s', s''\}$. Each college has one available seat. Let $S_a = \{s\}$, $S_b = \emptyset$, $S_c = \{s'\}$ and $S_d = \{s''\}$. Colleges a, b and, d are restrictive and c is unrestrictive, i.e., $C^R = \{a, b, d\}$. The preferences of colleges and students are given* *as follows:*

P_s	$P_{s'}$	$P_{s''}$	P_a	P_b	P_c	P_d
(s,c,I)	(s',b,I)	(s'',c,I)	(s,a,I)	(s,b,I)	(s',c,I)	(s'',d,I)
(s,c,W)	(s,b,W)	(s'',c,W)	(s,a,W)	(s,b,W)	(s',c,W)	(s'',d,W)
(s,b,I)	(s',a,I)	(s'',d,I)	(s',a,I)	(s',b,I)	(s'',c,I)	$\vdots$
(s,b,W)	(s',c,I)	$\vdots$	(s',a,W)	(s',b,W)	(s,c,I)	
(s,a,I)	$\vdots$		$\vdots$	$\vdots$	(s'',c,W)	
$\vdots$					(s,c,W)	

Suppose there exists a stable matching μ. Then, under μ students s, s', and s'' are assigned to contracts weakly better than (s,b,W), (s',c,I), and (s'',d,I), respectively. As a result, s'' cannot be assigned to (s'',c,I) under μ because no other student would be matched to d with the term I. If $\mu_s = (s,c,I)$, then s' needs to be assigned to (s',a,I). However, this matching is blocked by s' and b with contract (s',b,W). If $\mu_s = (s,c,W)$, then s'' needs to be assigned to (s'',d,I). However, this matching is blocked by s'' and c with contract (s'',c,W). If $\mu_s = (s,b,I)$ or $\mu_s = (s,b,W)$, then s' and s'' need to be assigned to (s',a,I) and (s'',c,W), respectively. However, this matching is blocked by s and c with contract (s,c,I). Hence, there is no stable matching under this problem.

In Example 5, when all colleges are restrictive, we illustrate that requiring the elimination of blocking coalitions including multiple colleges and students might result in the non-existence of a stable matching in some markets.

Example 5 *There are three colleges, $C = \{a,b,c\}$, and two students, $S = \{s,s'\}$. Each college is restrictive and has one available seat. Let $S_a = \{s\}$, $S_b = \{s'\}$, and $S_c = \emptyset$. The preferences of colleges and students are given as:*

P_s	$P_{s'}$	P_a	P_b	P_c
(s,c,I)	(s',c,I)	(s',a,I)	(s,b,I)	(s,c,I)
(s,b,I)	(s',c,W)	(s,a,I)	(s',b,I)	(s,c,W)
(s,c,W)	(s',a,I)	(s',a,W)	(s,b,W)	(s',c,I)
(s,b,W)	(s',a,W)	(s,a,W)	(s',b,W)	(s',c,W)
$\vdots$	$\vdots$	$x_\emptyset$	$x_\emptyset$	$x_\emptyset$

There is a unique stable matching in this market: $\mu = \{(s,c,W),(s',a,W)\}$. However, this matching can be blocked with a coalition composed of s, s', a, and b, such that the matching obtained from satisfying this blocking coalition as follows: $\nu = \{(s,b,I),(s',a,I)\}$. This matching ν is uncontested and is a Pareto improvement for all students. However, ν is blocked by student-college pair (s',c) with term W, i.e., ν is not stable.

In Example 6, we show how the UCOP mechanism's outcome is found.

Example 6 *There are seven colleges $C = \{a, b, c, d, e, f, g\}$. Each college has one available seat (i.e., $q_k = 1$ for all $k \in C$). Let $S_a = \{i\}$, $S_b = \{j\}$, $S_c = \{k\}$, $S_d = \{\ell\}$, $S_e = \{m\}$, $S_f = \{n\}$, and $S_g = \emptyset$. The preferences of colleges and students are given as follows:*

P_i	P_j	P_k	P_ℓ	P_m	P_n	P_a	P_b	P_c	P_d	P_e	P_f	P_g
(i, b, I)	(j, a, I)	(k, a, I)	(ℓ, f, I)	(m, d, I)	(n, c, I)	(k, a, I)	(i, b, I)	$\vdots$	$\vdots$	$\vdots$	(n, f, I)	(k, g, I)
(i, a, I)	(j, b, I)	(k, g, I)	(ℓ, e, I)	(m, d, W)	(n, f, I)	(j, a, I)	(j, b, I)				$\vdots$	(k, g, W)
$\vdots$	$\vdots$	(k, a, W)	(ℓ, f, W)	$\vdots$	$\vdots$	(i, a, I)	$\vdots$					$\vdots$
		(k, g, W)	(ℓ, e, W)			$\vdots$						
		$\vdots$	$\vdots$									

Stage 1: Initialization

We set $q^I_x = q^A_x = 1$ for all $x \in C \setminus \{g\}$, $q^I_g = 0$, and $q^A_g = 1$. For each $\widetilde{c} \in C$, we construct $\overline{P}_{\widetilde{c}}$ by moving contract $(\widetilde{s}, \widetilde{c}, I)$ to the top, where $\widetilde{s} \in S_{\widetilde{c}}$, e.g. (i, a, I) and (j, b, I) are ranked at the top under $\overline{P}_a$ and $\overline{P}_b$, respectively. We set $\overline{P}_s = P_s$ for all $s \in S$.

Stage 2: The COP Stage

	a	b	c	d	e	f	g
Step 1:	(j, a, I), (k, a, I)	(i, b, I)	(n, c, I)	(m, d, I)		(ℓ, f, I)	
Step 2:	(k, a, I)	(i, b, I),(j, b, I)	(n, c, I)	(m, d, I)		(ℓ, f, I)	
Step 3:	(i, a, I),(k, a, I)	(j, b, I)	(n, c, I)	(m, d, I)		(ℓ, f, I)	
Step 4:	(i, a, I)	(j, b, I)	(n, c, I)	(m, d, I)		(ℓ, f, I)	(k, g, I)
Step 5	(i, a, I),(k, a, W)	(j, b, I)	(n, c, I)	(m, d, I)		(ℓ, f, I)	
Step 6	(i, a, I)	(j, b, I)	(n, c, I)	(m, d, I)		(ℓ, f, I)	(k, g, W)

Stage 3: Stability Check

The obtained matching is contested. We remove (m, d, I) from m's preferences and return to Stage 2.

Stage 2: The COP Stage

	a	b	c	d	e	f	g
Step 1:	(j, a, I), (k, a, I)	(i, b, I)	(n, c, I)	(m, d, W)		(ℓ, f, I)	
Step 2:	(k, a, I)	(i, b, I),(j, b, I)	(n, c, I)	(m, d, W)		(ℓ, f, I)	
Step 3:	(i, a, I),(k, a, I)	(j, b, I)	(n, c, I)	(m, d, W)		(ℓ, f, I)	
Step 4:	(i, a, I)	(j, b, I)	(n, c, I)	(m, d, W)		(ℓ, f, I)	(k, g, I)
Step 5	(i, a, I),(k, a, W)	(j, b, I)	(n, c, I)	(m, d, W)		(ℓ, f, I)	
Step 6	(i, a, I)	(j, b, I)	(n, c, I)	(m, d, W)		(ℓ, f, I)	(k, g, W)

Stage 3: Stability Check

The obtained matching is contested. We remove (ℓ, f, I) *from* ℓ*'s preferences and return to Stage 2.*

Stage 2: The COP Stage

	a	b	c	d	e	f	g
Step 1:	(j,a,I), (k,a,I)	(i,b,I)	(n,c,I)	(m,d,W)	(ℓ,e,I)		
Step 2:	(k,a,I)	(i,b,I),(j,b,I)	(n,c,I)	(m,d,W)	(ℓ,e,I)		
Step 3:	(i,a,I),(k,a,I)	(j,b,I)	(n,c,I)	(m,d,W)	(ℓ,e,I)		
Step 4:	(i,a,I)	(j,b,I)	(n,c,I)	(m,d,W)	(ℓ,e,I)		(k,g,I)
Step 5	(i,a,I),(k,a,W)	(j,b,I)	(n,c,I)	(m,d,W)	(ℓ,e,I)		
Step 6	(i,a,I)	(j,b,I)	(n,c,I)	(m,d,W)	(ℓ,e,I)		(k,g,W)

Stage 3: Stability Check

The obtained matching is contested. We remove (n, c, I) *from* n*'s preferences and return to Stage 2.*

Stage 2: The COP Stage

	a	b	c	d	e	f	g
Step 1:	(j,a,I), (k,a,I)	(i,b,I)		(m,d,W)	(ℓ,e,I)	(n,f,I)	
Step 2:	(k,a,I)	(i,b,I),(j,b,I)		(m,d,W)	(ℓ,e,I)	(n,f,I)	
Step 3:	(i,a,I),(k,a,I)	(j,b,I)		(m,d,W)	(ℓ,e,I)	(n,f,I)	
Step 4:	(i,a,I)	(j,b,I)		(m,d,W)	(ℓ,e,I)	(n,f,I)	(k,g,I)
Step 5	(i,a,I),(k,a,W)	(j,b,I)		(m,d,W)	(ℓ,e,I)	(n,f,I)	
Step 6	(i,a,I)	(j,b,I)		(m,d,W)	(ℓ,e,I)	(n,f,I)	(k,g,W)

Stage 3: Stability Check

The obtained matching is contested. We remove (ℓ, e, I) *from* ℓ*'s preferences and return to Stage 2.*

Stage 2: The COP Stage

	a	b	c	d	e	f	g
Step 1:	(j,a,I), (k,a,I)	(i,b,I)		(m,d,W)		(n,f,I), (ℓ,f,W)	
Step 2:	(k,a,I)	(i,b,I),(j,b,I)		(m,d,W)	(ℓ,e,W)	(n,f,I)	
Step 3:	(i,a,I),(k,a,I)	(j,b,I)		(m,d,W)	(ℓ,e,W)	(n,f,I)	
Step 4:	(i,a,I)	(j,b,I)		(m,d,W)	(ℓ,e,W)	(n,f,I)	(k,g,I)
Step 5	(i,a,I),(k,a,W)	(j,b,I)		(m,d,W)	(ℓ,e,W)	(n,f,I)	
Step 6	(i,a,I)	(j,b,I)		(m,d,W)	(ℓ,e,W)	(n,f,I)	(k,g,W)

Stage 3: Stability Check

The obtained matching is stable. We go to Stage 4.

Stage 4: Welfare Improvement

There is one cycle: $m \to d \to \ell \to e \to m$. *We implement the cycle to produce the following matching:*

a	b	c	d	e	f	g
(i, a, I)	(j, b, I)		(m, d, I)	(ℓ, e, I)	(n, f, I)	(k, g, W)

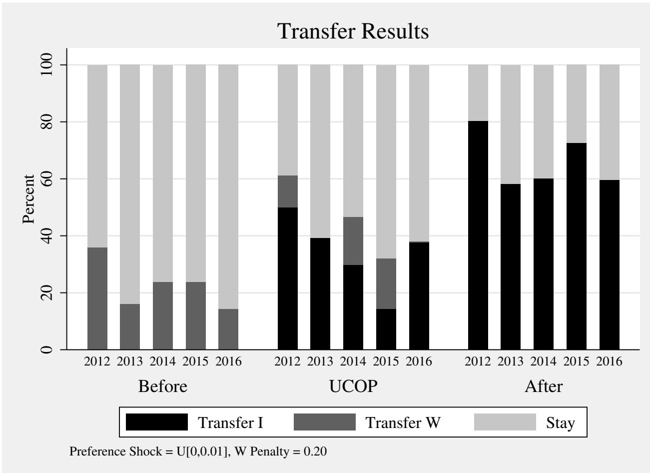

Figure A1: Transfer Results

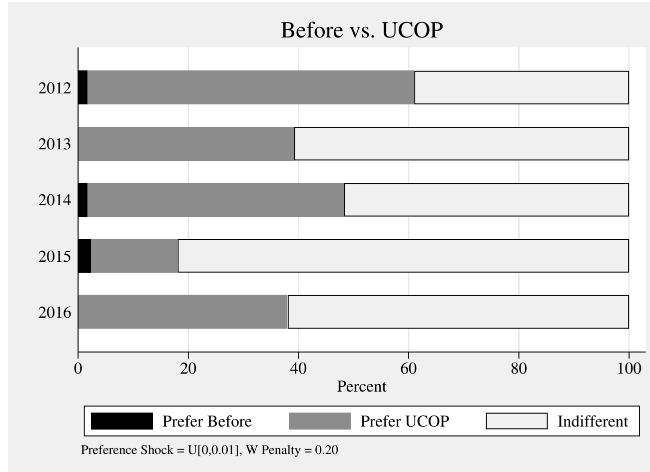

Figure A2: Students' Preferences: Before vs. UCOP

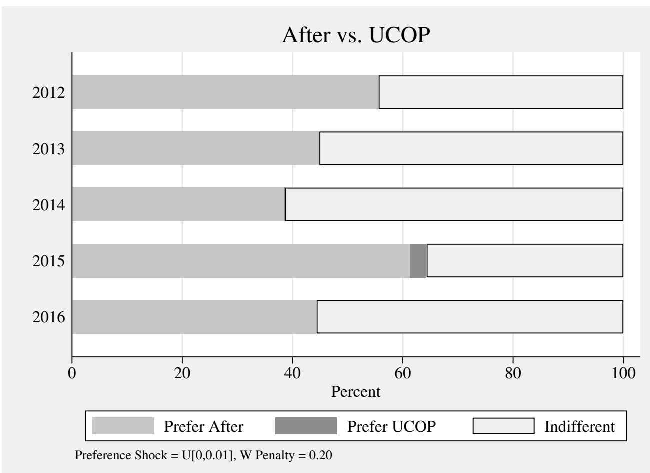

Figure A3: Students' Preferences: After vs. UCOP

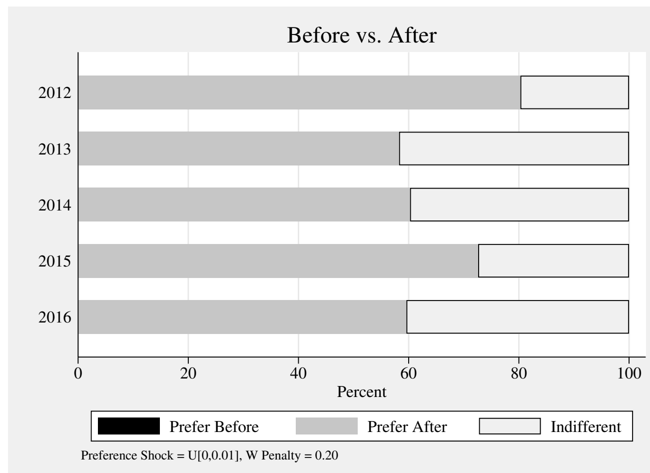

Figure A4: Students' Preferences: Before vs. After

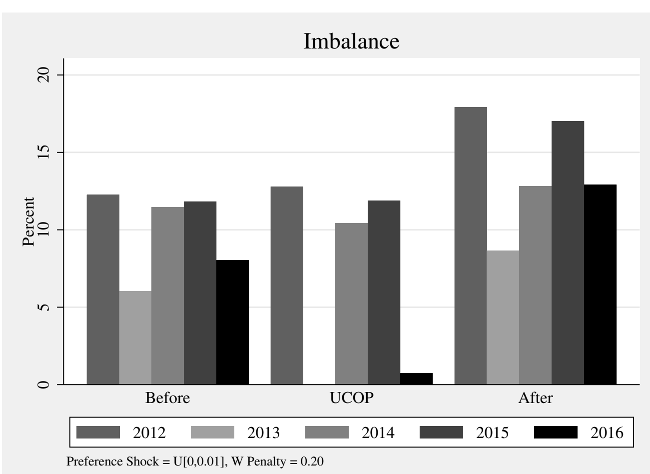

Figure A5: Colleges' Imbalance

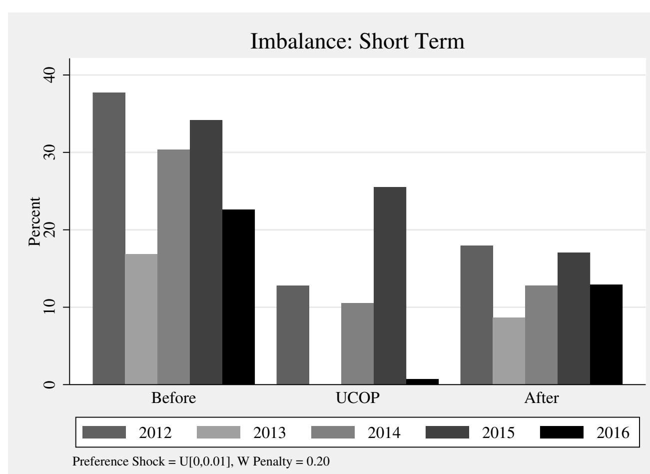

Figure A6: Colleges' Short-Term Imbalance

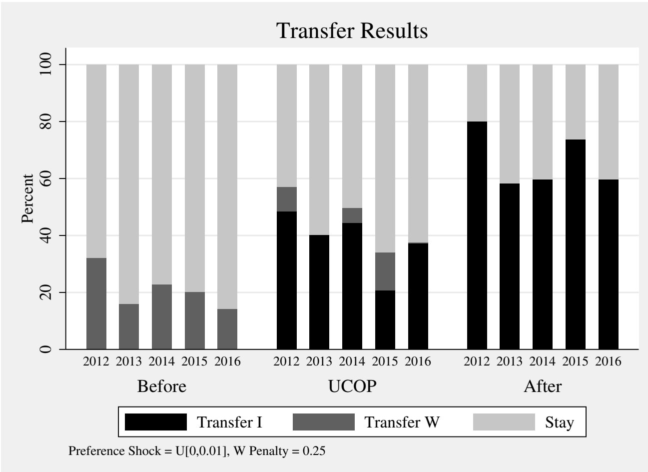

Figure A7: Transfer Results

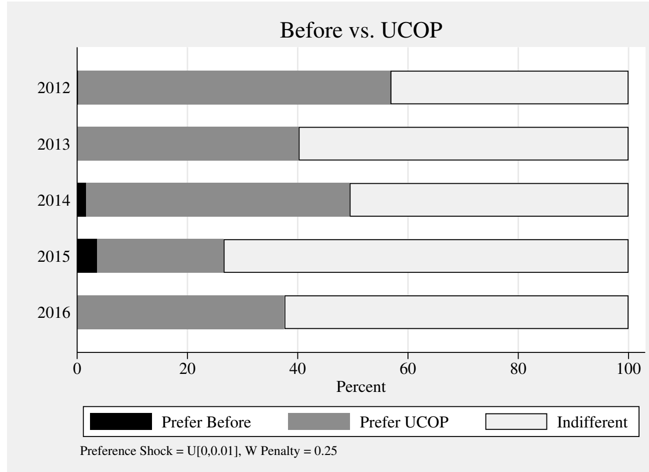

Figure A8: Students' Preferences: Before vs. UCOP

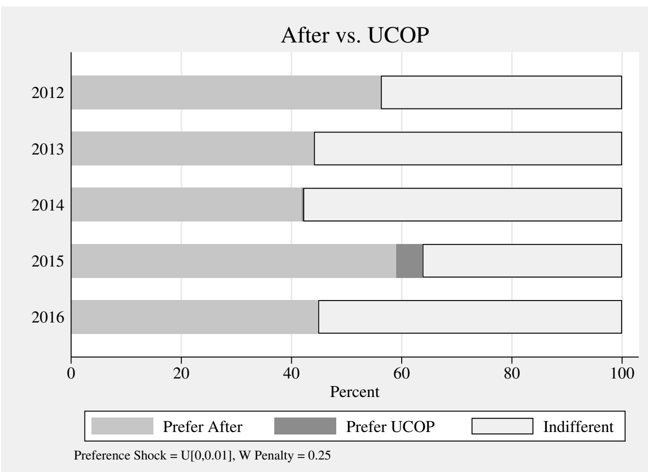

Figure A9: Students' Preferences: After vs. UCOP

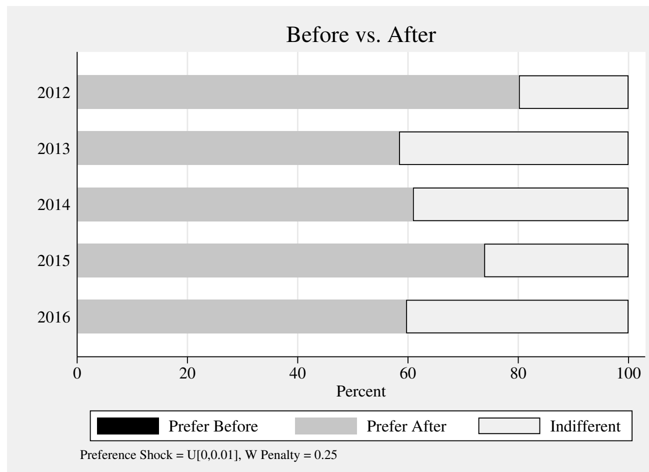

Figure A10: Students' Preferences: Before vs. After

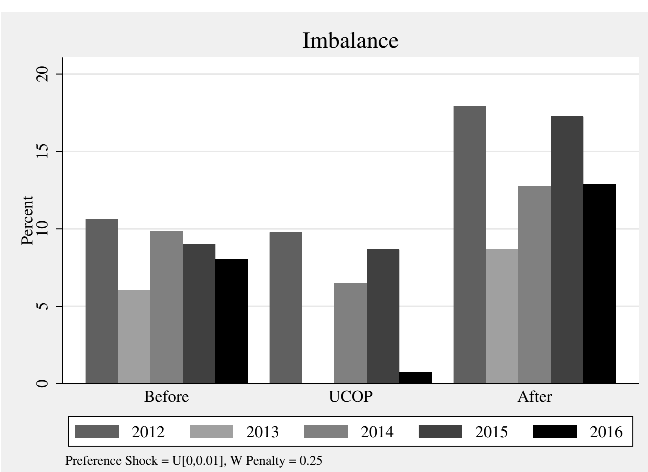

Figure A11: Colleges' Imbalance

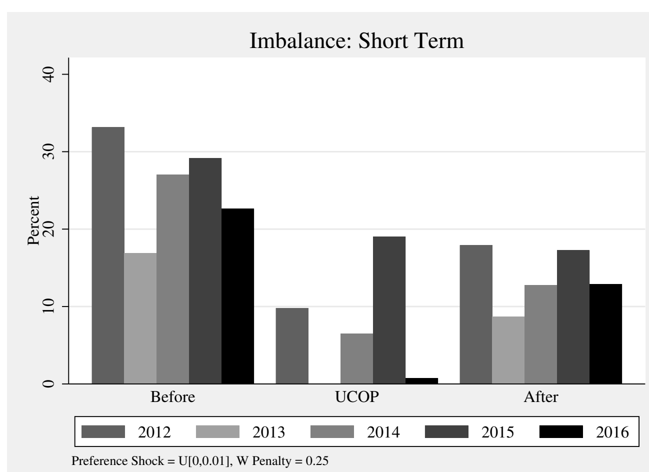

Figure A12: Colleges' Short-Term Imbalance

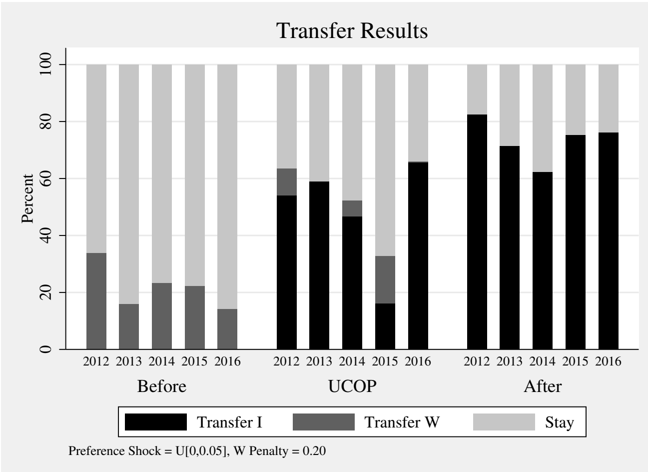

Figure A13: Transfer Results

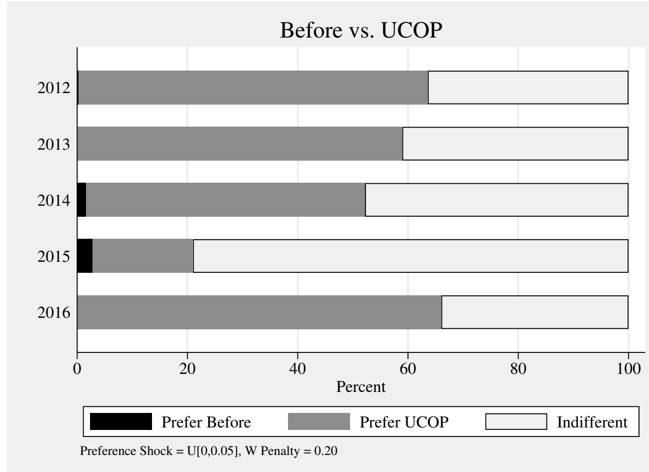

Figure A14: Students' Preferences: Before vs. UCOP

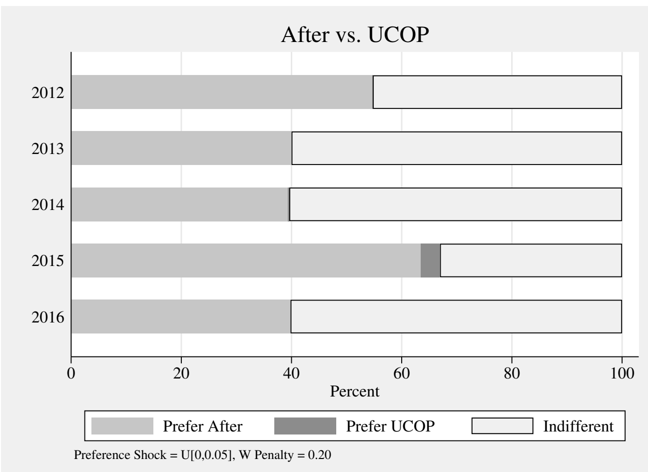

Figure A15: Students' Preferences: After vs. UCOP

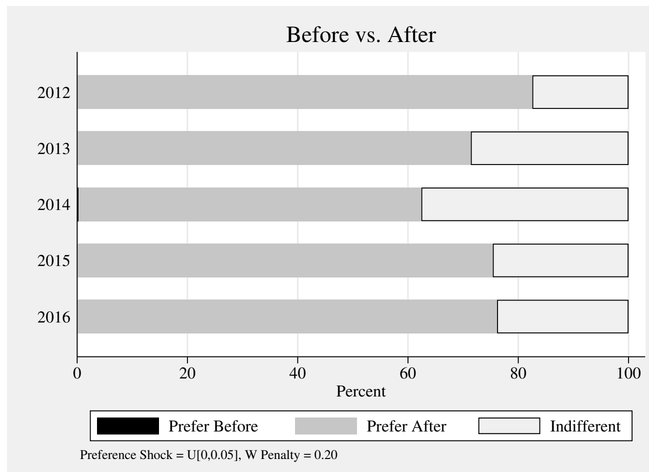

Figure A16: Students' Preferences: Before vs. After

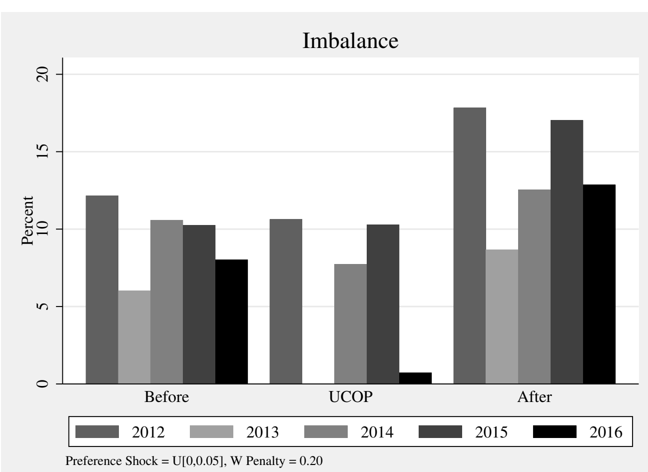

Figure A17: Colleges' Imbalance

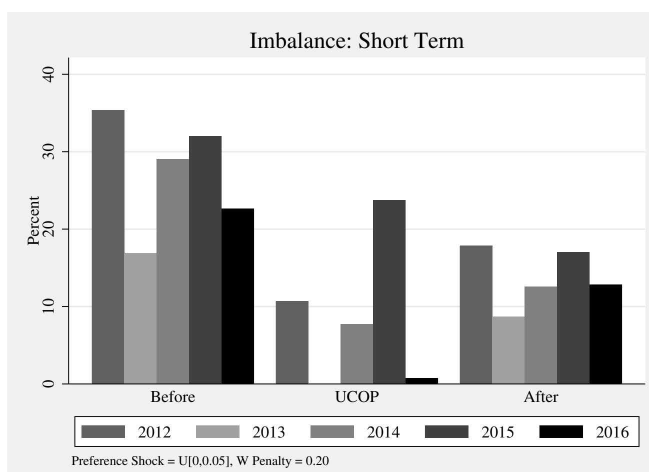

Figure A18: Colleges' Short-Term Imbalance

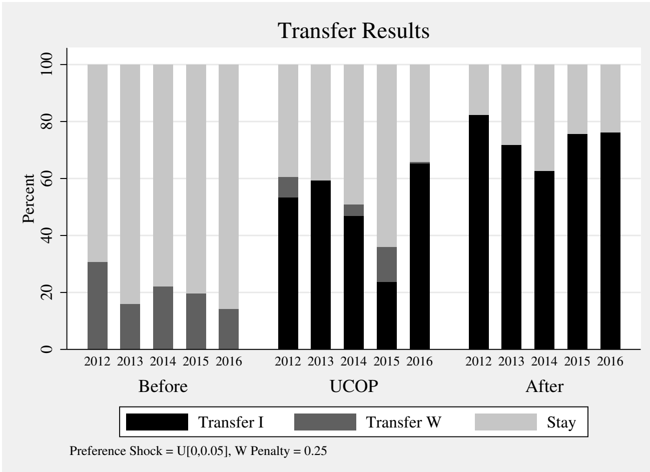

Figure A19: Transfer Results

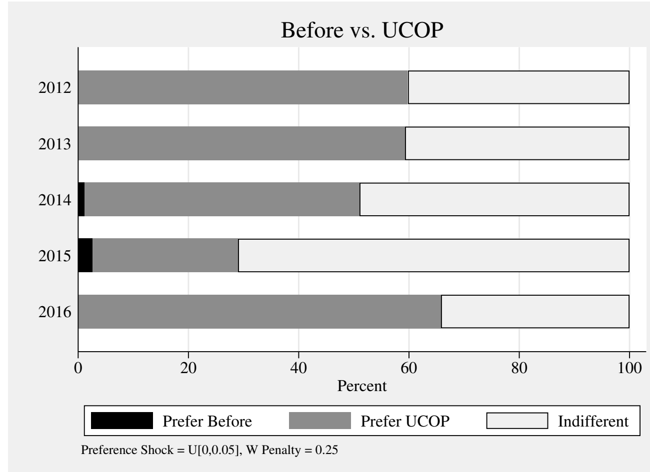

Figure A20: Students' Preferences: Before vs. UCOP

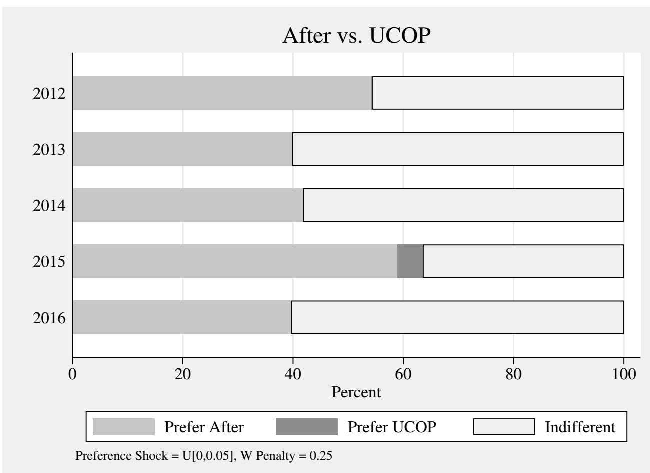

Figure A21: Students' Preferences: After vs. UCOP

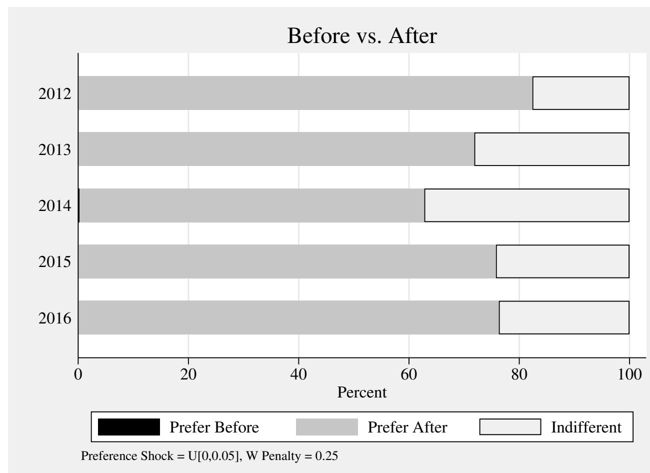

Figure A22: Students' Preferences: Before vs. After

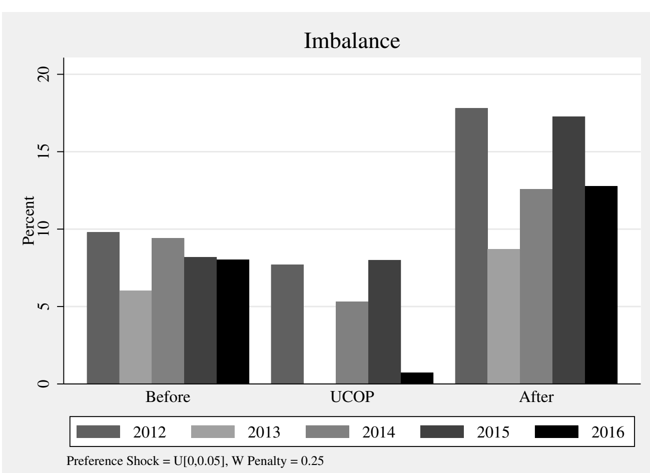

Figure A23: Colleges' Imbalance

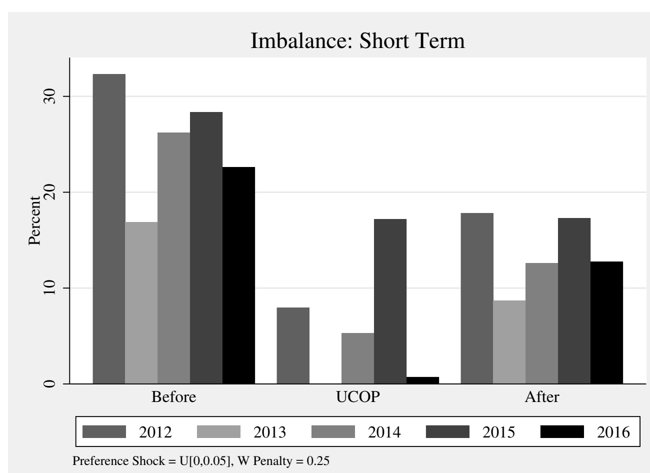

Figure A24: Colleges' Short-Term Imbalance

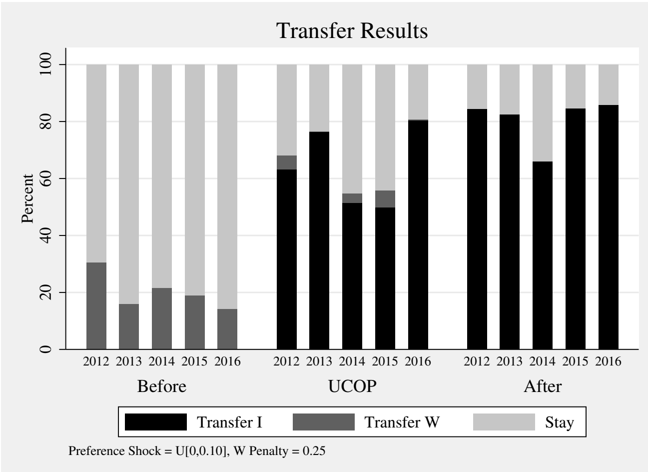

Figure A25: Transfer Results

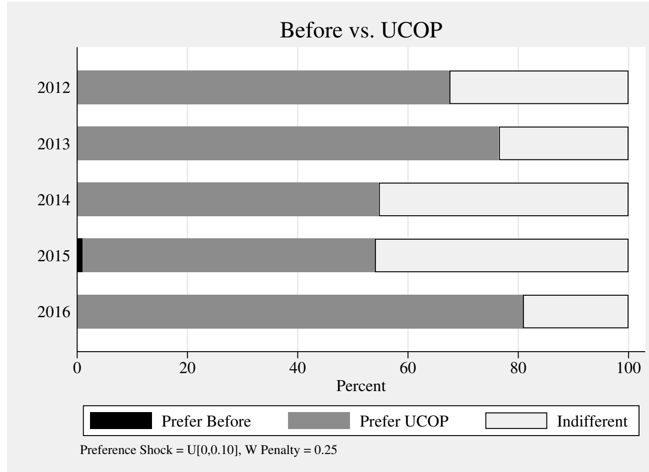

Figure A26: Students' Preferences: Before vs. UCOP

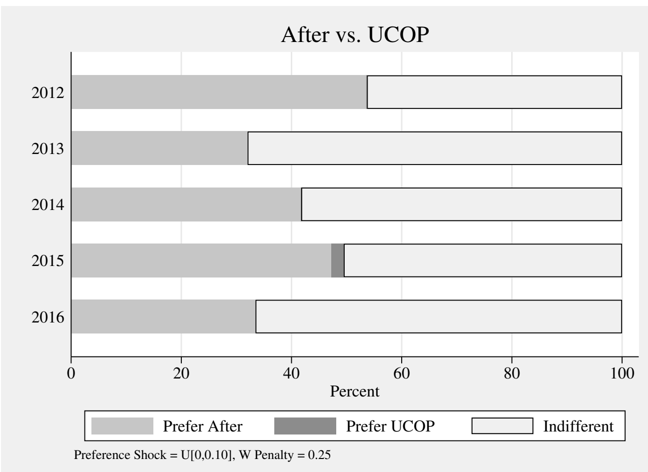

Figure A27: Students' Preferences: After vs. UCOP

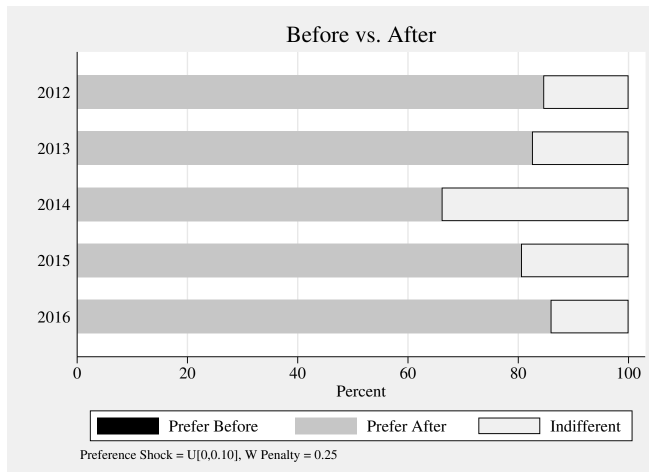

Figure A28: Students' Preferences: Before vs. After

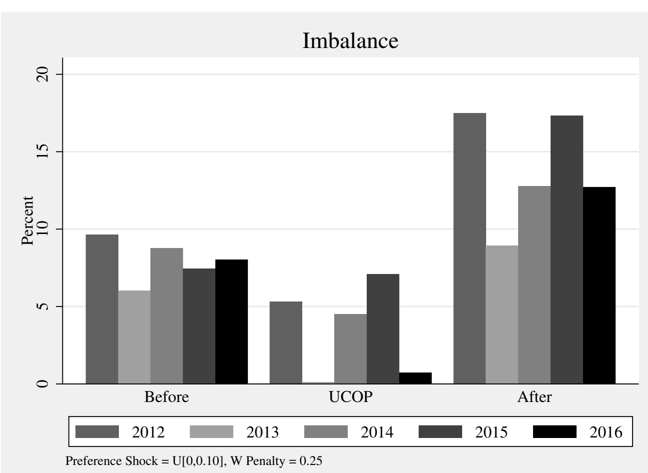

Figure A29: Colleges' Imbalance

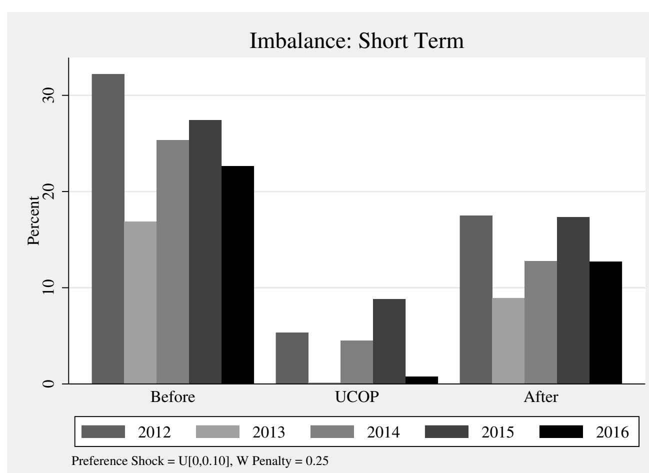

Figure A30: Colleges' Short-Term Imbalance

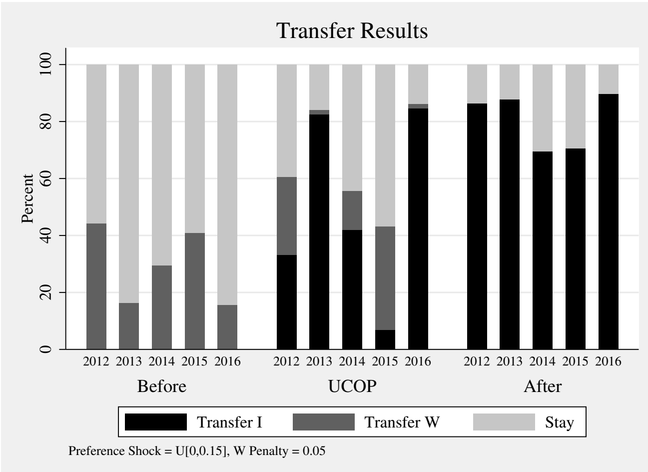

Figure A31: Transfer Results

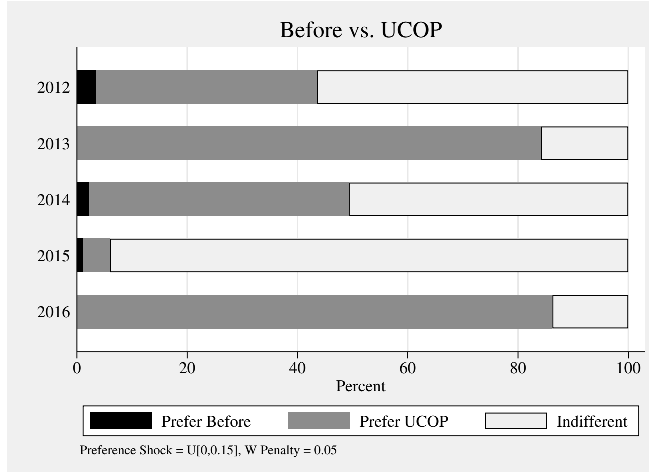

Figure A32: Students' Preferences: Before vs. UCOP

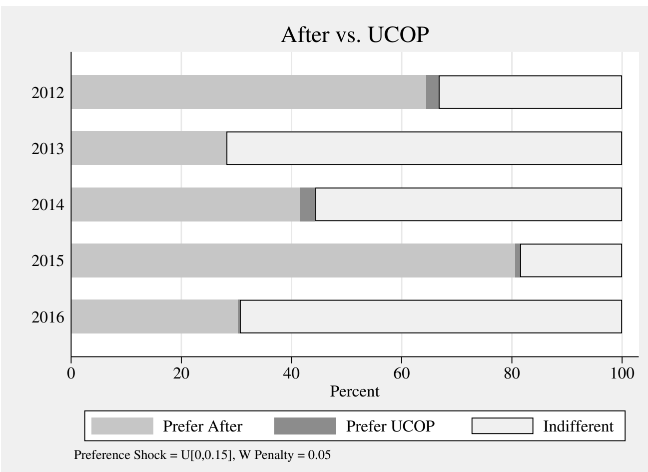

Figure A33: Students' Preferences: After vs. UCOP

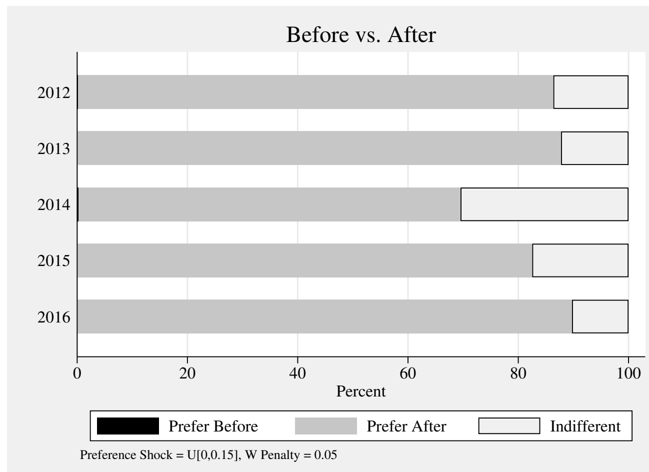

Figure A34: Students' Preferences: Before vs. After

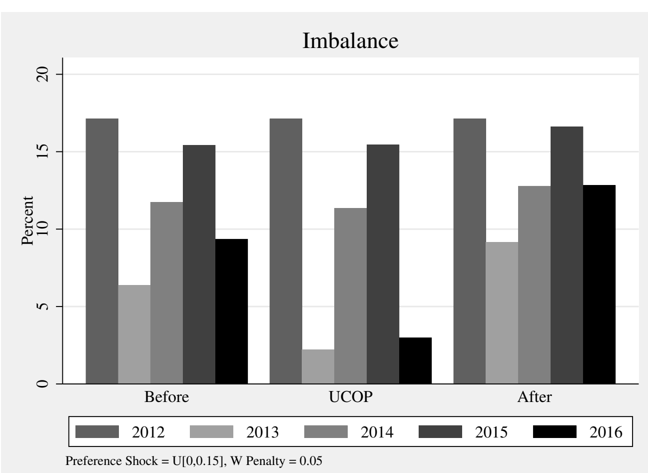

Figure A35: Colleges' Imbalance

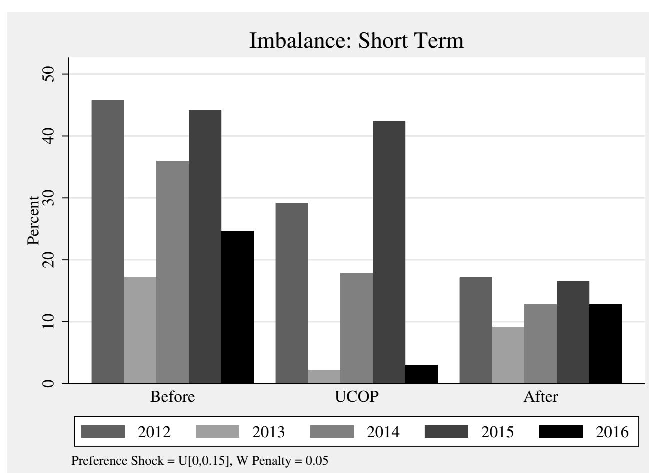

Figure A36: Colleges' Short-Term Imbalance

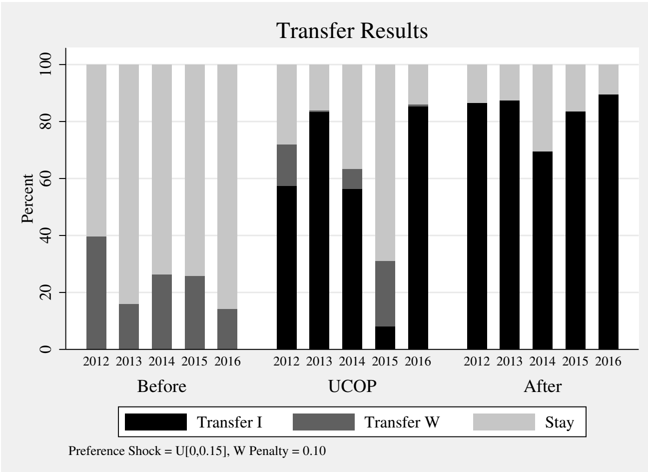

Figure A37: Transfer Results

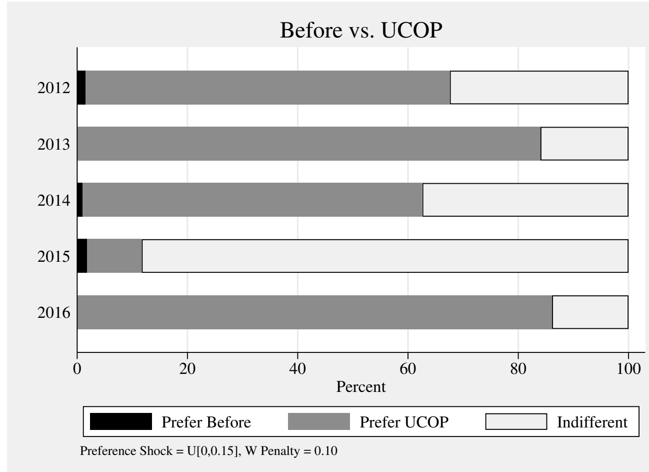

Figure A38: Students' Preferences: Before vs. UCOP

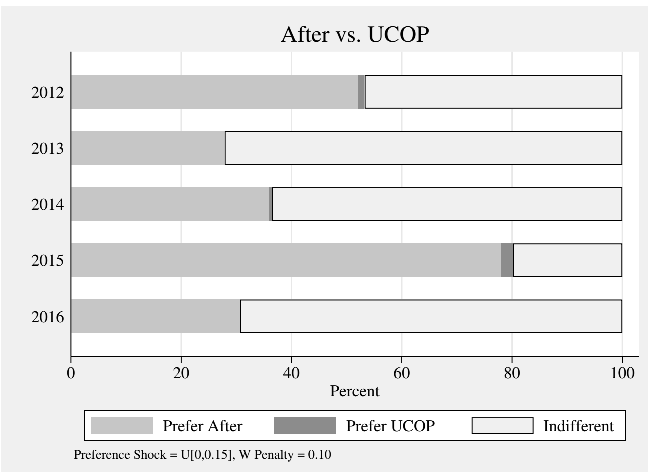

Figure A39: Students' Preferences: After vs. UCOP

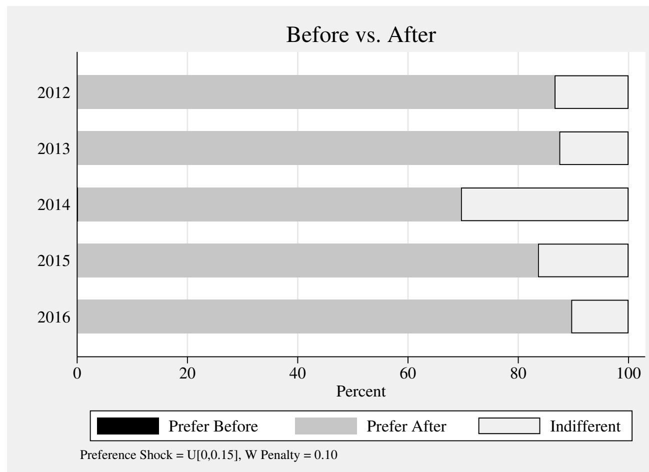

Figure A40: Students' Preferences: Before vs. After

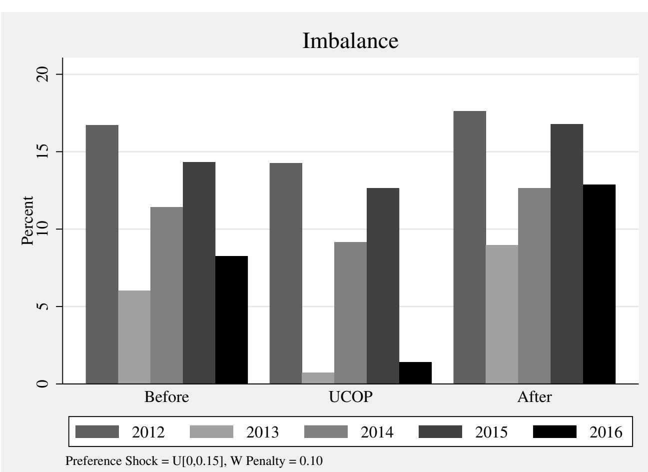

Figure A41: Colleges' Imbalance

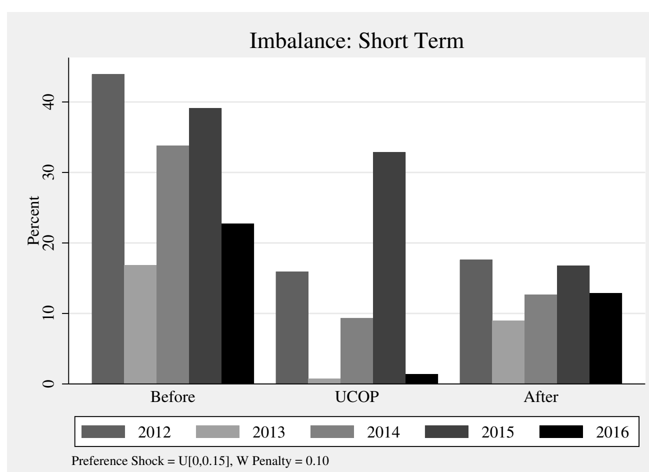

Figure A42: Colleges' Short-Term Imbalance

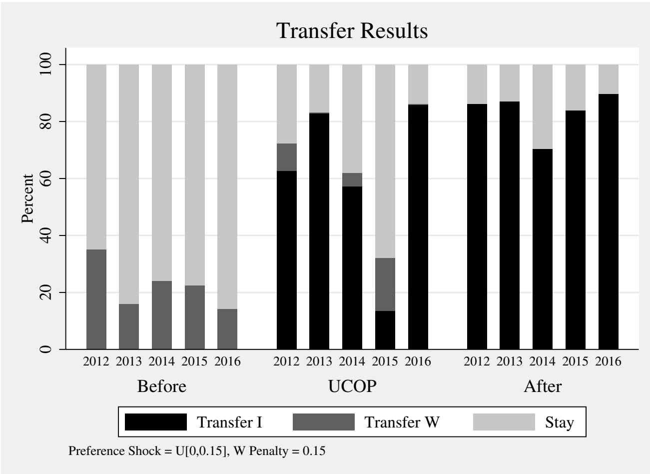

Figure A43: Transfer Results

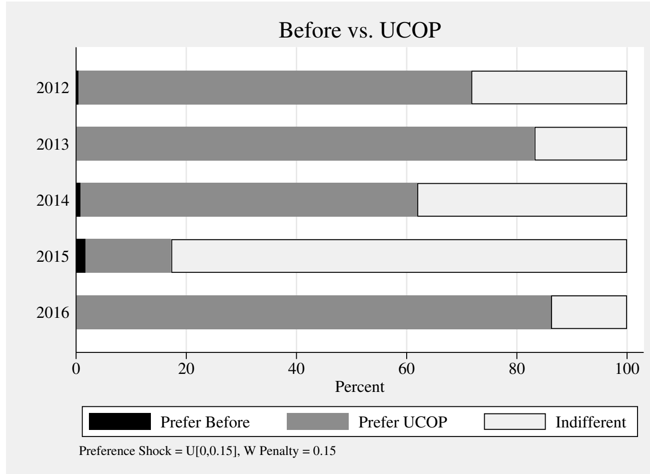

Figure A44: Students' Preferences: Before vs. UCOP

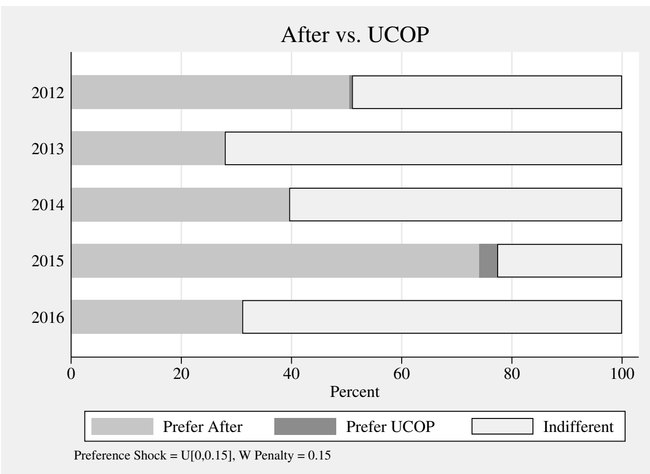

Figure A45: Students' Preferences: After vs. UCOP

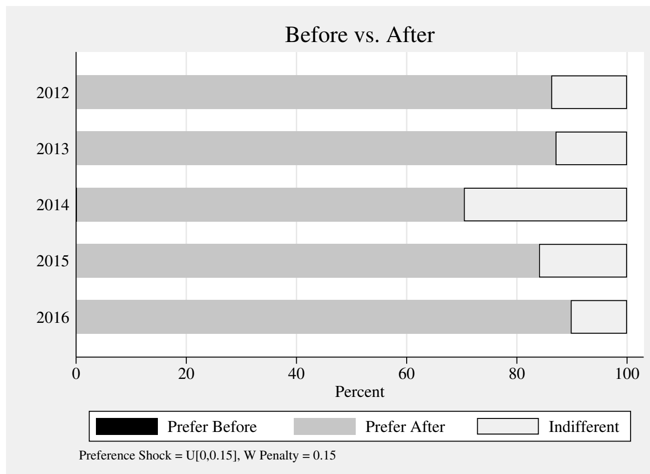

Figure A46: Students' Preferences: Before vs. After

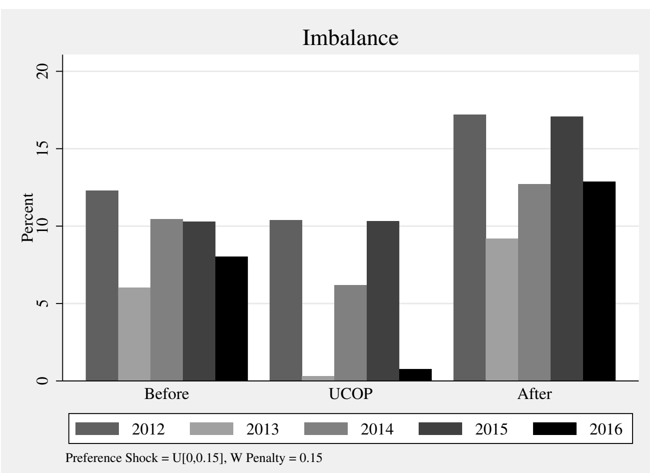

Figure A47: Colleges' Imbalance

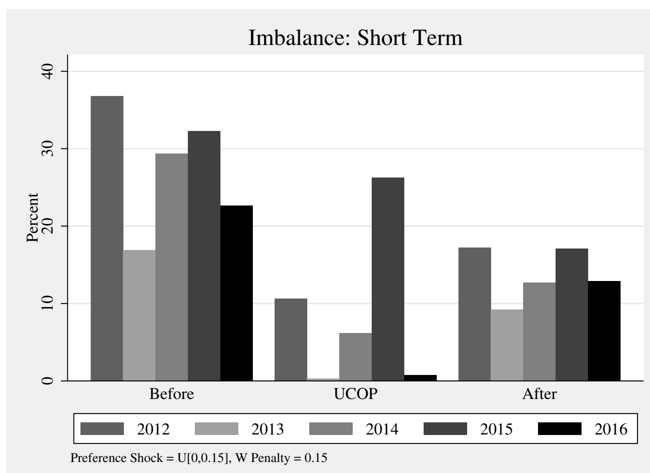

Figure A48: Colleges' Short-Term Imbalance

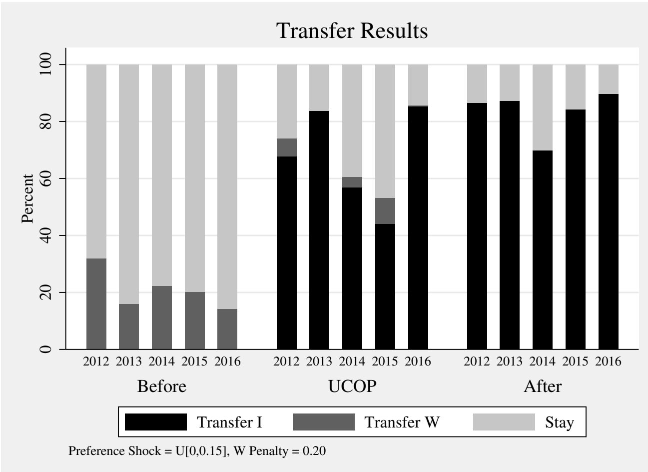

Figure A49: Transfer Results

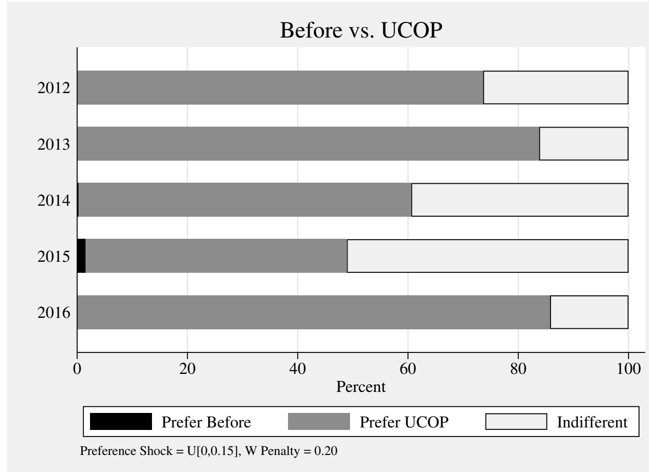

Figure A50: Students' Preferences: Before vs. UCOP

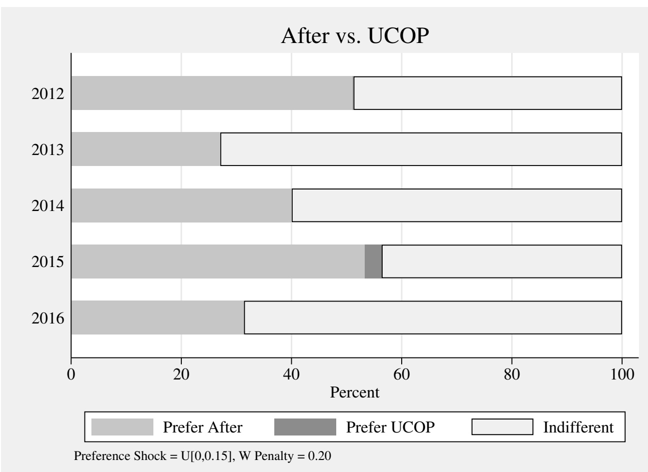

Figure A51: Students' Preferences: After vs. UCOP

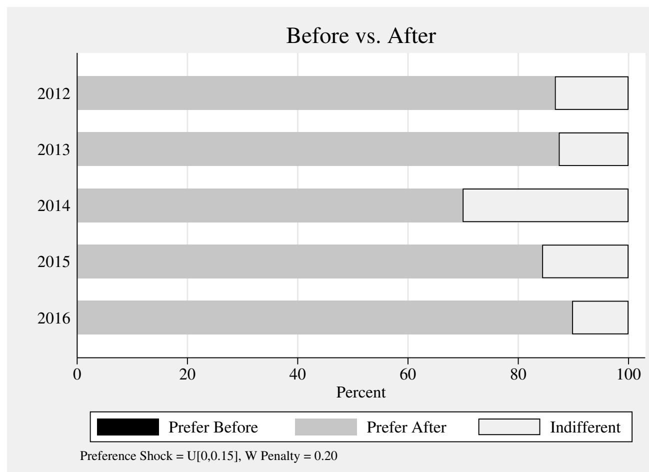

Figure A52: Students' Preferences: Before vs. After

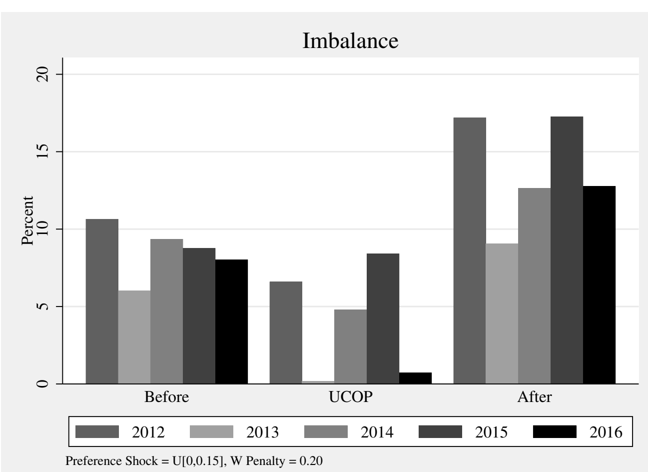

Figure A53: Colleges' Imbalance

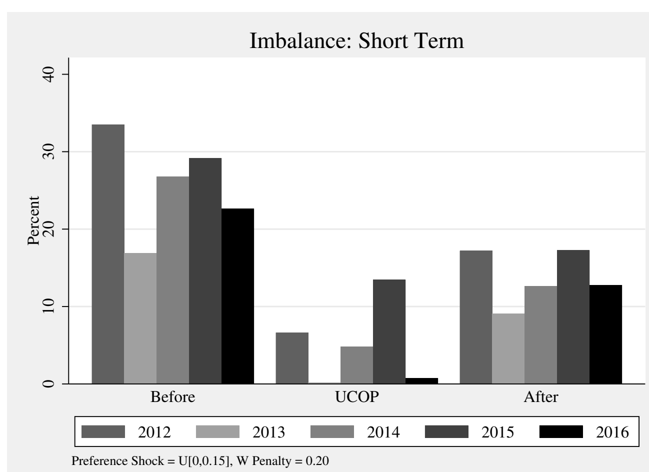

Figure A54: Colleges' Short-Term Imbalance